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The Mathematics Department believes its students must learn the importance of mathematics, the integration of different branches of mathematics, the application of math to real-life problems, and the connections between mathematics and other disciplines. This course is concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications.

Since this course is not designed to prepare students for the Advanced Placement tests, the main concepts and their applicable methods and applications are emphasized.

Knowing algebra is a fundamental prerequisite to success in higher mathematics. In this unit, we will review those algebra topics that are essential in the study of calculus.

Grade: 11-12	**Calculus** **Algebra Reference**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	The skills used and mastered in algebra are critical to the successful study of calculus.
Essential Questions	What are the main algebra concepts and skills needed in calculus?
Content Standard:	25.1.3.9.1 Students will model and solve problems with linear, quadratic and absolute value equations; and linear inequalities. 25.1.3.9.2 Students will determine equivalent representations of an algebraic equation or inequality to simplify and solve problems.

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Performance Expectations (Student outcomes)	Content Polynomial operations (add, subtract, multiply) Factoring (GCF, difference of perfect squares, trinomials, perfect cubes) Rational expressions (simplify, add, subtract, multiply) Solving equations (linear, quadratic, rational) Inequalities (linear, quadratic, rational) Exponents (simplify by using the laws) Radicals (add, subtract, multiply, divide, rationalize) Skills Simplify polynomial and rational expressions Factor polynomials Solve equations and inequalities Simplify radical and exponential expressions

Strategies/Modes (examples)	**Materials/Resources (examples)**	**Assessments (examples)**
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter R, Sections R1 to R7	• homework assignments • quizzes • tests • alternative assessments

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In this unit, linear functions are reviewed. Slopes and intercepts, two of the recurring themes of calculus, are found and used in their most basic form while studying linear functions. The relationship of variables is explored as well as using linear functions to make predictions.

Grade: 11-12	**Calculus** **Linear Functions**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions 25.4 Working with Data
Enduring Understanding	A line shows the relationship between two sets of data. Lines of best fit are used to predict outcomes from data and real life situations.
Essential Questions	What is a linear relationship between two variables? What does a line of best fit represent in a given situation? How is it used in conjunction with real world situations?
Content Standard:	25.1.1.9.1 Students will identify, describe, create, and generalize numeric, geometric, and statistical patterns with tables, graphs, words, and symbolic rules. 25.1.1.9.2 Students will make and justify predictions based on patterms. 25.1.2.9.3 Students will recognize and explain the meaning of the slope and x- and y-intercepts as they relate to a context, graph, table, or equation. 25.1.2.9.4 Students will evaluate and interpret the graphs of linear, exponential, and polynomial functions. 25.1.2.9.6 Students will recognize the effect of changes in parameters on the graphs of functions or relations. 25.4.1.9.2 Students will develop, use, and explain applications and limitations of linear and non-linear models and regression in a variety of contexts. 25.4.1.9.5 Students will recognize the limitations of mathematical models based on sample data as representations of real-world situations. 25.4.2.9.1 Students will estimate an unknown value between data points on a graph (interpolation) and make

	predictions by extending the graph (extrapolation).
Performance Expectations (Student outcomes)	Content Lines - intercepts - slope - graphs - equations Systems of linear equations Least-squares lines - correlation - using as a method of predicting Applications - cost - revenue - profit - supply - demand - equilibrium price Skills Identify the characteristics of lines and graph linear functions Write equations of lines Determine the least-squares equation that fits a set of data Find and interpret the correlation coefficient for a set of data Solve systems of linear equations

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Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 1, Sections 1.1 to 1.3	• homework assignments • quizzes • tests • alternative assessments

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In this unit, the general properties of functions will be studied. The special nature of specific families of functions, such as quadratic, polynomial, rational, exponential, and logarithmic functions will be investigated. This knowledge will be used to analyze and understand a large number of real-life applications. In addition, knowing when the use of one function is more appropriate than another will be stressed, as well as predicting how the dependent variable will react to changes in the independent variable.

Grade: 11-12	**Calculus** **Functions**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	Functions are a major tool for describing the real world in mathematical terms.
Essential Questions	What are the general shapes of the graphs of different functions? How are different classes of functions used to describe real world applications?
Content Standard:	25.1.1.9.1 Students will identify, describe, create, and generalize numeric, geometric, and statistical patterns with tables, graphs, words, and symbolic rules. 25.1.1.9.3 Students will identify the characteristics of functions and relations including domain and range. 25.1.1.9.5 Students will describe and compare properties and classes of functions including exponential, polynomial, rational, logarithmic, and trigonometric. 25.1.1.9.6 Students will analyze essential relations in a problem to determine possible functions that could model the situation. 25.1.2.9.1 Students will represent functions and relations on the coordinate plane. 25.1.2.9.4 Students will evaluate and interpret the graphs of linear, exponential, and polynomial functions. 25.1.2.9.5 Students will relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes, and line of symmetry for that function. 25.1.2.9.6 Students will recognize the effect of changes in parameters on the graphs of functions or relations. 25.1.3.9.5 Students will combine, compose, and invert functions.

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Performance Expectations (Student outcomes)	Content Functions - domain/range - from graphs - from equations - evaluate - properties (operations) of functions Identifying functions - vertical line test - equations - mapping Quadratics - vertex - axis of symmetry - intercepts - graph Graphs of polynomial functions - classify - even/odd functions Rational functions - define - asymptotes - vertical - horizontal - graph Exponential Functions - solving equations - same base - raising to a power - graph Applications of functions

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	- average (cost, profit, etc.) - cost-benefit analysis - interest - growth/decay - effective rate - present value - half-life <u>Skills</u> Identify whether a relation is a function Determine the domain/range of a function Evaluate functions Apply the properties of functions Determine the parts of a quadratic function and graph the parabola Identify the asymptotes and intercepts of rational functions Graph polynomial and rational functions Solve exponential equations Change between exponential and logarithmic forms Evaluate logarithmic expressions Apply the properties of logarithms and solve logarithmic equations

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 2, Sections 2-1 to 2-6	• homework assignments • quizzes • tests • alternative assessments

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The concept of limit is one of the ideas that distinguish calculus from algebra and trigonometry. In this unit, the idea of a limit is defined. Values of the limits of functions are calculated using substitution, graphical investigation, numerical approximation, algebra, or some combination of these. One of the uses of limits is to test functions for continuity. Continuous functions arise frequently in scientific work because they model such an enormous range of natural behavior.

Grade: 11-12	**Calculus** **Limits and Continuity**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	The concept of a limit is the fundamental building block on which all other calculus concepts are based.
Essential Questions	What is a limit? What does it mean for a function to be continuous on an interval? What are the basic types of discontinuity?
Content Standard:	25.1.1.9.11 Students will apply the concepts of limits to sequences and asymptotic behavior of functions.
Performance Expectations (Student outcomes)	Content Definition of a limit Finding limits - from a graph - using substitution - one-sided - two-sided - at infinity - using the rules of limits

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	Definition of continuity Finding points of discontinuity - from a graph - algebraic functions - piecewise functions Solving applications <u>Skills</u> Identify the limit from the graph of a function Explain when limits do and do not exist Apply the rules of limits Find the value of a limit by substitution Determine limits at infinity Solve real-world applications involving limits Identify points of discontinuity from a graph Find values of x for which a function is discontinuous Solve real-world applications involving the concept of continuity

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 3, Sections 3.1, 3.2	• homework assignments • quizzes • tests • alternative assessments

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In this unit, the definition of the derivative is explored – first as a special limit, then interpreted as the slope of the tangent line, instantaneous rate of change, and instantaneous velocity. Because the definition of the derivative contains the phrase "provided this limit exists", the various ways that a derivative can fail to exist will be explored. Once the definition is understood, a set of rules will be developed for which the derivative of a wide range of functions can be quickly and easily calculated without having to directly apply the definition of the derivative each time.

Grade: 11-12	**Calculus** **Calculating Derivatives**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	The derivative is a mathematical tool that is used to study and describe rates of change.
Essential Questions	How is a function's differentiability at a point related to its continuity? What is the relationship between the average rate of change of a function and its instantaneous rate of change? How does the rate of change of one variable affect another variable?
Content Standard:	25.1.1.9.11 Students will apply the concepts of limits to sequences and asymptotic behavior of functions. 25.1.2.9.5 Students will relate the graphical representation of a function to its functions family and find equations, intercepts, maximum or minimum values, asymptotes, and line of symmetry for that function. 25.1.2.9.7 Students will recognize that the slope of the tangent line to a curve represents the rate of change.
Performance Expectations (Student outcomes)	<u>Content</u> Rates of change - average - instantaneous Applications of rates of change Derivatives - definition

- limit of the difference quotient
- graphical differentiation

Calculating derivatives
- constants
- powers
- products
- quotients
- chain rule
- exponential (#^x)
- e^x
- logarithmic
- ln

Tangents
- finding slope
- equations of tangent lines

Applications
- marginal
- straight line motion
- average (cost, profit, revenue)

<u>Skills</u>

Define and find an average rate of change
Apply the formula for instantaneous rate of change
Define a derivative
Use the limit definition to find derivatives
Calculate derivatives of various functions
Write the equation of the tangent line to a graph at a specific point
Apply derivatives to a variety of real-world problems

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Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 3, Sections 3.3, 3.4 Chapter 4, Sections 4.1 to 4.5	• homework assignments • quizzes • tests • alternative assessments

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In this unit, the derivatives of a function are used to determine definitive characteristics of a function. Intervals where the function increases/decreases, relative extrema, intervals of concavity, and points of inflections are all explored by using derivatives and the derivative tests. Once all of these pieces are found for a particular function, an accurate graphical representation can be constructed.

Grade: 11-12	**Calculus** **Curve Sketching**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	Derivatives and derivative tests are used to accurately sketch graphs of functions.
Essential Questions	What do the derivatives of a function tell you about the shape of its graph? What are the First and Second Derivative Tests of a function and how do they help in sketching its graph? How does an inflection point affect the shape of a graph?
Content Standard:	25.1.2.9.5 Students will relate the graphical representation of a function to its functions family and find equations, intercepts, maximum or minimum values, asymptotes, and line of symmetry for that function. 25.1.2.9.7 Students will recognize that the slope of the tangent line to a curve represents the rate of change.
Performance Expectations (Student outcomes)	<u>Content</u> Characteristics of graph - increasing/decreasing - concavity - inflection points - relative extrema Higher order derivatives Derivative tests - first - second

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	<u>Skills</u> Determine points of relative extrema Identify intervals for which a function is increasing/decreasing Calculate higher derivatives State intervals of concavity and identify inflection points Graph the function

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 5, Sections 5.1 to 5.4	• homework assignments • quizzes • tests • alternative assessments

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In this unit, several applications of the derivative are explored. The concept of absolute (as opposed to relative) extrema is investigated and then applied to a variety of real world applications. The technique of differentiating a function that is written in an implicit form is explored and then used to solve related rate problems, in which one or more rates are given and another is to be found. Finally, using the differential to find linear approximations to functions is studied.

Grade: 11-12	**Calculus** **Applications of the Derivative**
CSDE Standard	25.1 Algebraic Reasoning: Patterns and Functions
Enduring Understanding	Derivatives are used to solve problems in a variety of situations.
Essential Questions	What is the strategy for solving related rates problems? What is the difference between absolute and relative extrema? How are absolute extrema used to solve real world problems?
Content Standard:	25.1.2.9.5 Students will relate the graphical representation of a function to its functions family and find equations, intercepts, maximum or minimum values, asymptotes, and line of symmetry for that function. 25.1.2.9.7 Students will recognize that the slope of the tangent line to a curve represents the rate of change.
Performance Expectations (Student outcomes)	<u>Content</u> Absolute extrema - graphs - equations - application problems Implicit differentiation and related rates Linear approximation <u>Skills</u>

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	Identify points of absolute extrema Solve problems using absolute extrema Differentiate functions that are written implicitly Use implicit differentiation to solve related rate problems Estimate quantities using linear approximation

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 6, Sections 6.1, 6.2, 6.4, 6.5, 6.6	• homework assignments • quizzes • tests • alternative assessments

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This unit and the next, deal with integration, another main idea in calculus. There are two aspects of integration. The first is indefinite integration, or finding an antiderivative. In the beginning of the unit, the techniques used to find the antiderivative will be explored, as well as their relationship to the techniques used to find derivatives. The second aspect is definite integration, which can be used to find the area under a curve. The Fundamental Theorem of Calculus unites these two ideas by showing that the way to find the area under a curve is to use an antiderivative.

Grade: 11-12	**Calculus** **Integrals**
CSDE Standard	25.3 Geometry and Measurement
Enduring Understanding	The Fundamental Theorem of Calculus is the concept that unifies the two branches of calculus – differential and integral.
Essential Questions	How is the antiderivative defined? What is the difference between a definite and an indefinite integral? What are the various methods used to find an antiderivative?
Content Standard:	25.3.3.9.1 Students will select appropriate units, scales, degree of precision , and strategies to determine length, angle measure, perimeter, circumference, and area of plane geometric figures. 25.3.3.9.4 Students will use two dimensional representations, formal and informal methods to solve surface area and volume problems.
Performance Expectations (Student outcomes)	<u>Content</u> Definition of antiderivative Calculating integrals - powers - constants - sum/difference

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	- exponential functions - logarithmic functions - substitution - by parts Definite integrals - area definition - Fundamental Theorem of Calculus <u>Skills</u> Define an antiderivative Calculate indefinite integrals Use the Fundamental Theorem of Calculus to find definite integrals Use integrals to approximate areas under the curve

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 7, Sections 7.1 to 7.4	• homework assignments • quizzes • tests • alternative assessments

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Now that the concept of finding antiderivatives have been explored, several applications of integration will be studied. The idea of a definite integral can not only be used to find the area under a curve, but it can be used to find the area bounded by two different curves. When finding an antiderivative is not feasible, two different rules for numerical integration – the trapezoidal rule and Simpson's rule – can be used. In addition to area, integration can also be used to find volumes of revolution. Finally the concept of improper integrals (those having infinity as one of the bounds) will be discussed.

Grade: 11-12	**Calculus** **Applications of Integration**
CSDE Standard	25.3 Geometry and Measurement
Enduring Understanding	Integration is the process used to find the area bounded by curves.
Essential Questions	How are integrals related to the concept of "area"? How can definite integrals be used to find area? How do you define and calculate the area of the region between the graphs of two continuous functions? How do you define and calculate the volumes of solids by the method of slicing?
Content Standard:	25.3.3.9.1 Students will select appropriate units, scales, degree of precision , and strategies to determine length, angle measure, perimeter, circumference, and area of plane geometric figures. 25.3.3.9.4 Students will use two dimensional representations, formal, and informal methods to solve surface area and volume problems.
Performance Expectations (Student outcomes)	<u>Content</u> Area between two curves - a curve and x-axis - two curves Numerical Integration

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	- trapezoidal rule - Simpson's Rule Volume - shell method - disk method Improper integrals - converge/diverge - evaluate <u>Skills</u> Use integrals to calculate the areas of regions in a plane Use integrals to calculate the volumes of solids of revolution Approximate values of definite integrals by using the trapezoidal or Simpson's rule Determine whether an improper integral converges or diverges Evaluate improper integrals that converge

Strategies/Modes (examples)	Materials/Resources (examples)	Assessments (examples)
• Guided practice • Worksheets • Homework • Cooperative Group work • Quizzes • Tests • Projects • Math Labs	Calculus with Applications, Brief Version (Lial, Greenwell, Ritchey) Chapter 7, Sections 7.5, 7.6 Chapter 8, Sections 8.1, 8.2, 8.4	• homework assignments • quizzes • tests • alternative assessments