Grade/Subject	Grade 6/ Mathematics
Unit Title	Unit 2: Operating with Positive Rational Numbers
Overview of Unit	This unit will explore the number system through applying and extending previous understandings of multiplication and division to divide fractions by fractions, compute fluently with multi-digit numbers that include decimals and solve real-world and mathematical problems.
Pacing	14 days

Background Information For The Teacher
The Operating with Positive Rational Numbers unit displays a shift in thinking for sixth grade from a computation emphasis to a critical understanding of why these computations work and are applied. The critical areas of focus for Grade 6 in Operating with Positive Rational Numbers include students: • using the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense using these operations to solve problems, • extending their previous understandings of number sense and the ordering of numbers to the full system of rational numbers, In fifth grade, students will have: • fluently multiplied multi-digit whole numbers using the standard algorithm, • found whole-number quotients and solved word problems of whole numbers (up to four-digit dividends and two-digit divisors), • applied and extended previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions, In this unit, 6th grade students will: • interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem, • fluently divide multi-digit numbers using the standard algorithm,

- fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation,

Teachers will need to have a strong understanding of computation of whole numbers, decimals and fractions and calculating volume. Teachers must have a strong grasp on the CCSS's standard algorithm for division of whole numbers (demonstrated in the Explanations and Examples section below). Teachers should be aware that students may enter the unit with varying levels of experience and expertise with computation, specifically with fractional components. Student centered activities and student-to-student discourse is essential to build on and foster a greater student understanding of these concepts critical to real-world applications. In this unit, all the Standards for Mathematical Practices will be addressed. Specifically, students will: make strong relationships between the concepts by making sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, model with mathematics and look for and make use of structure.

Essential Questions (and Corresponding Big Ideas)	
How do operations illustrate the relationships between numbers? • Operations show ways that numbers can be decomposed and recomposed. How do we solve real world problems involving fractions and decimals? • We use multiple strategies including symbols, pictures and models to represent solutions. How does knowing the relationship between divisors and quotients help me become a better mathematician? • Knowing the inverse relationship between divisors and quotients helps me estimate and know if my work is reasonable.	
Core Content Standards	**Explanations and Examples**
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using	**6.NS.1.** • Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution. **Context:** You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt

visual fraction models and equations to represent the problem.

For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3 (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

This standard emphasizes the use of fraction models including manipulatives and visual diagrams to interpret, represent, and solve word problems with division of fractions. Students write equations to show how word problems are solved. Sixth graders interpret the meaning of fractions, the meaning of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. What they are actually doing is working with a complex fraction. In the example $\frac{2}{3} \div \frac{3}{4}, \frac{2}{3}$ is the numerator and $\frac{3}{4}$ is the denominator as $\frac{\frac{2}{3}}{\frac{3}{4}}$.

What the **teacher** does:

- Begin teaching division of fractions with a concrete hands-on model such as pattern blocks and students can touch and move. For example provides a yellow hexagon, a red trapezoid, a blue rhombus, and green triangles to each group of 4 students. Ask students to solve the problem $\frac{1}{2} \div \frac{1}{6}$.

  Use pattern block's red trapezoid to show ½.

from a snack pack.

How much of the recipe can you make?

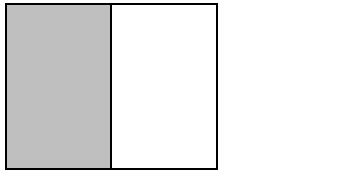
Explanation of Model:

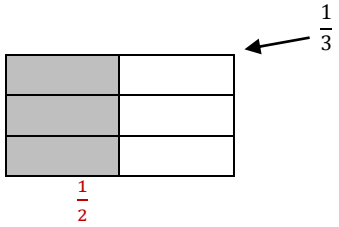
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

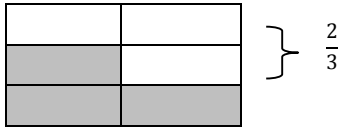
The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model. $\frac{2}{3}$ is the new referent unit (whole).

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make ¾ of the recipe.







a whole ½ 1/3 1/6

Ask students how many 1/6s fit on the red trapezoid. Ask student to show $\frac{1}{2} \div \frac{1}{6} = 3$.

- Continue with additional visual drawings to help students understand quotients of fractions by building from familiar scenarios. For example, 3 people share ½ of a cake. How many pieces does each person get? Ask students to draw a diagram to show each person gets 1/6 of the cake.

- Ask students, "In the equation $\frac{1}{2} \div \frac{1}{6}$, why does 3 make sense as the answer when it is so much bigger than ½ or 1/6?" (Answer: 3 tells you how many 1/6 s make ½.)
- Present word problems for students to solve such as, " I have 2/3 of a yard of fabric and want to make pencil bags that are 1/6 of a yard each. How many can I make?" Have students create number line drawings to illustrate $\frac{2}{3} \div \frac{1}{6} =$.
  - Students should start with a number line divided into thirds.
  - Divide each third into halves to create sixths.
  - Each circled part represents 1/6. There are 4 sixths in 2/3. I can make 4 pencil bags.
- Lead students from using manipulatives and diagrams for dividing fractions to computing answers procedurally using the multiplicative inverse, which is a synonym for "reciprocal." With the number x, its reciprocal (or multiplicative inverse) is 1/x. Try the following problems:
  - You have 5/8 pound of nuts. You decide to share a ¼ of a pound with each of your friends. How many friends can you

- Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt from a snack pack.

How much of the recipe can you make?

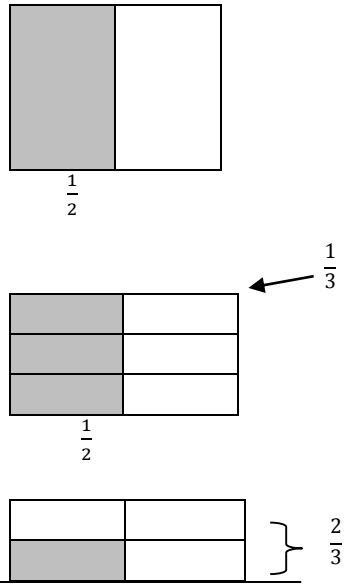
Explanation of Model:

The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model. $\frac{2}{3}$ is the new referent unit (whole).

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make ¾ of the recipe.



share the nuts with?
- You have a ¾ acre lot. You must divide it into 3/8 acre lots. How many lots will you create?

- Incorporate the following vocabulary terms into instruction about division of fractions: numerator, denominator, reciprocal, quotient, and multiplicative inverse.
- Ensure students have opportunities to talk with the teacher and each other to make sense of what they ware learning about division of fractions. Talk about the relationship between multiplication and division, specifically explaining that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ because ¾ of 8/9 is 2/3.

6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.

The focus for this standard is using the traditional, standard algorithm for long division. However, major emphasis is placed on the meaning of division and the understanding of place value of multi-digit numbers when dividing fluently. Fluently dividing multi-digit numbers means dividing quickly and accurately. To have fluency, students need sufficient, on-going practice with long division.

What the **teacher** does:

- Pose problems solving situations to focus on the understand and meaning of division such as, "Sam wants to purchase a new smart phone for $168. He earns $12 an hour for mowing lawns. How many hours will he need to mow lawns to have enough money for his phone?"
- Explore division with a variety of models that are used as a tool for division such as the area model or partial quotients model. Students describe their understanding of place value as they divide such as when dividing 7,964 by 44. In this example, as students write a 1 in the quotient, they should say, "There are 100 forty-fours in 7964."
- Investigate mental math to quickly estimate the quotient of a division problem. Try using compatible numbers to make the estimation much easier. The division problem 73 divided by 23 could be thought of as 75 divided by 25. Explore with additional division problems using compatible

What the **teacher** does:

- Model division of fraction with manipulatives, visual diagrams)bar models, number lines), and word problems.
- Divide fractions procedurally using multiplicative inverse.
- Interpret what the quotient represents in mathematical and real-world problems.
- Understand that multiplication and division are inverse operations.

Misconceptions and Common Errors:

Sixth graders may incorrectly model division of fractions. Some students may think dividing by ½ is the same as dividing in half. Dividing y ½ means to find how many one-halves there are in a quantity. Dividing in half means to take quantity and divide it into two equal parts. To address the misconception, ask them to demonstrate two examples, one that show dividing by ½ and another that shows dividing in half. For example, 9 divided by ½ equals 18 and 9 divided in half equals 4 ½.

6.NS.2. Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level.

numbers to estimate the reasonableness of answers.

- Model the following vocabulary terms associated with multi-digit division: division, dividend, divisor, quotient and algorithm.
- Provide cyclical, distributed practice over time to continually review multi-digit division throughout the year as fluency develops over time. Allow students to practice in a variety of ways including pencil and paper algorithms, mental math, and with problem solving situations.

As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200 thirty-twos in 8456 " and could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	There are 60 thirty twos in 2056.

6.NS.3. Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation. This standard requires students to extend the models and strategies for the four operations previously developed for whole numbers in Grades 1-5 to decimals. Emphasis for addition, subtraction, multiplication, and division of multi-digit decimals is on using standard algorithms. Students estimate answers and self-correct errors in computation if needed. Fluency adding, subtracting, multiplying, and dividing multi-digit decimals means students can find a sum, difference, product, or quotient quickly and accurately. To obtain fluency, students need sufficient, on-going practice for each operation.	26 32)8456 −6400 2056 −1920 136	60 times 32 is 1920. 2056 minus 1920 is 136.	
	264 32)8456 −6400 2056 −1920 136 −128	There are 4 thirty twos in 136. 4 times 32 is 128.	
	264 32)8456 −6400 2056 −1920 136 −128 8	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8	

What the **teacher** does:

- Model estimating the sum, difference, product, or quotient from the problems before performing the operation.
- Explore estimating the sum and then finding the exact sum for adding decimals. For example, to estimate 15.2 and 7.65, an estimate of the sum could be 15 + 7 or 22. Expect that students know if their estimate is too high or too low. Ask students to reason, "Why does it make sense that your answer must be larger than 22?" Teach students to use their estimates to self-correct any errors in their computation.
- Practice previous understanding related to the patterns involved when multiplying and dividing by powers of 10 to develop fluency with operations with multi-digit decimals,
- Ensure that students understand the role of place value in the operations of addition, subtraction, multiplication, and division. Use decimal blocks to review place value if needed.
- Focus on the following vocabulary terms associated with decimal computation: estimate, multi-digit decimals, and algorithm.
- Provide cyclical, distributed practice over time to continually practice multi-digit decimal computation throughout the year as fluency develops over time. Allow students to practice in a variety of ways, including pencil-and-paper algorithms, Mental math, and with problem-solving situations.

What the **students** do:

- Communicate the meaning of division using precise mathematical vocabulary.
- Understand place value of multi-digit numbers and use it when dividing.
- Know division is the inverse of multiplication.
- Develop fluency with the traditional, standard algorithm for division of multi-digit whole numbers.
- Use compatible numbers to estimate the reasonableness of answers.

Misconceptions and Common Errors:

For some students, the traditional standard division algorithm is difficult simply because of the many steps involved in the procedure. Some sixth graders may focus on individual digits when dividing rather than thinking about the whole number. Others may ignore place value and get an incorrect answer. To help students, remind them to describe both the place value as they divide and place value of the digits in the quotients. Ask them to show the steps of division, one at a time. Provide graph papers to keep the work legible.

6.NS.3. The use of estimation strategies supports student understanding of operating on decimals.

Example:

- First estimate the sum and then find the exact sum of 14.4 and 8.75.

An estimate of the sum would be 14 + 9 or 23. Students could also state if their estimate is low or high, they would expect their answer to be greater than 23. Students can use their estimates to self-correct.

Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths) whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy-five hundredths fit together to make one whole and 25 hundredths.

Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.

Examples:

- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF?

	Solution: $2^2 * 3 = 12$. Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor.) • What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM? Solution: $2^3 * 3 = 24$. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 (2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3). • Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know? • Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions. ○ 27 + 36 = 9 (3 + 4) 63 = 9 x 7 63 = 63 ○ 31 + 80 There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93. What the **students** do:

	• Understand decimal place values. • Know basic facts for addition, subtraction, multiplication, and division. • Add, subtract, multiply, and divide multi-digit decimals using the standard algorithms. • Use vocabulary associated with multi-digit computation with multi-digit decimals both orally and in writing. <u>Misconceptions and Common Errors:</u> Some students may not remember to use the concept of place value when adding tenths to hundredths. For example, when adding five-tenths to eighty-five hundredths, some students may not realize the answer is one whole and thirty-five hundredths. To help with this misconception, try using decimal blocks or drawing a picture to show how the decimals have been added. Adding a zero to 0.5 to write 0.50 before adding it to 0.85 helps students focus on the place values.
Standards for Mathematical Practice	**Explanations and Examples**
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. **6.NS.1,** This cluster focuses on the use of visual fraction models and equations to divide whole numbers by fractions and fractions by fractions. Sixth graders interpret the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. **MP1. Make sense of problems and persevere in solving them.** **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP6. Attend to precision.** **Compute fluency with multi-digit numbers and find common factors and multiples.** **6.NS.2, 6.NS.3** Fluency and accuracy with multi-digit addition, subtraction, and division is the focus for this cluster along with a spotlight on greatest common factors and least common multiples. The cluster also builds on previous leaning of the multiplicative structure as well as prime and	Sixth graders interpret and make sense of problem involving division of fractions. Students use the main of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Sixth graders use manipulatives to model everyday problems with fractions. Students communicate precisely with others and use clear mathematical language when discussing the understanding and procedure for dividing fractions.

composite numbers. **MP2. Reason abstractly and quantitatively.** **MP7. Look for and make use of structure.** **MP8. Look for and express regularity in repeated reasoning.**	 Students are able to understand the meaning of division problem. Sixth graders apply division algorithms to divide multi-digit numbers. Students consider the reasonableness of an estimated quotient.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
• division of fractions • division of multi-digit numbers • addition, subtraction, multiplication and division of multi-digit decimals • distributive property of two whole numbers 1-100 volume of a right rectangular prism with fractional edge lengths	• INTERPRET (quotients of fractions) • COMPUTE (quotients of fractions) • SOLVE (word problems involving division of fractions by fractions) • REPRESENT (problems using models and equations) • USE a standard algorithm to perform calculations with multi-digit decimals
UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*	
• Operations illustrate relationships between numbers. • Real world situations can be represented symbolically and pictorially. • Multiple strategies and models can be utilized to solve a variety of problems involving fractions and decimals. • Relative magnitude of a solution (quotient) depends on the size of the divisor**.**	

Common Student Misconceptions for this Unit
• Students may believe dividing by one-half is the same as dividing in-half. (Dividing by half means to define how many halves there are in one quantity, while dividing in-half means splitting the quantity into two equal parts.) • Students will often relate all multiplication products to be larger than their addends and all division quotients to be smaller. (Inverse relationships will exist with fractional and less-than-whole decimal values.) • When multiplying mixed number fractions, students will often multiply the whole-number parts together and then multiply the fractional parts together and put these products together to create a sum. • Students may confuse the placement of the decimal in different operations rather than being specific to the operation being performed.

Unit Assessment/Performance Task	DOK
Unit 2 Test Performance Task "Adjusting a Recipe"	

Vocabulary
Absolute Value Algorithm Approximate Common Factor Coordinate Plane Coordinates

Denominator
Distributive Property
Dividend
Divisor
Factors
Greatest Common Factor
Improper fraction
Integers
Inverse
Irrational number
Irrational Numbers
Least Common Multiple
Mixed number
Multi-Digit
Multiplicative Inverse
Numerator
Ordered Pairs
Prime Factorization
Prime Factorization
Quadrant
Quotient
Rational numbers
Reciprocal
Signed Numbers

Key Learning Activities/Possible Lesson Focuses (order may vary)

The following activities are broken into "lessons," even though each may take more or less than one class period depending on school schedule.

Lesson sequence

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Lesson 1: Multiplying Fractions

Use area models and pictorial representations.

Lesson 2: Divide Fractions

Students will compute the quotient with fractions. Provide dry-erase boards and markers to your students. Draw a fraction bar on the board with 3 of 4 parts shaded. Have students recreate your fraction bar on their boards. Ask students how they can divvy this bar up into eights. Have students use a different color to show their work. Last, have each student present their picture and explain the division sentence that would correspond.

Goals

- Develop ways to model quotients
- Develop an algorithm for dividing fractions
- Solve word problems using operations of fractions

Suggested Activities

- Students, in whole-class instruction or small groups, can represent division of fractions using manipulatives (such as freezer pops and candy bars) and models (such as drawings and squares) to develop an algorithm and solve problems.
- http://www.uen.org/Lessonplan/preview?LPid=15443
  With partners or individually, students will determine fractional lengths of an area with manipulatives (different size sticky notes).

Resources

- Connected Math 2, Bits and Pieces II
- Crosswalk Coach Mathematics Grade 6, Domain 1 Lesson 9
- Teaching the Common Core Math Standards, The Number System: 6.NS.1

Lesson 3: Operations with Multi-digit Decimals

Students will use standard algorithms to perform calculations with multi-digit decimals. Present your class with coupons for groceries. Coupons should vary in amount and product. Compose prices for the products on the coupons and display on the board. Ask students several questions such as: "How much is your item after using the coupon?" "The supermarket is doubling coupons on Tuesday. How much will your item be Tuesday?" "Purchase your product and your neighbor's product. How much would you pay for both items before and after using your coupons?"

Goals

- Estimate the results of operations on decimals and viable argument about why your estimation is valid
- Use knowledge of place value in working with decimals
- Know when to use each operation in a situation involving decimals
- Construct real world situations where people often choose to use decimals instead of common fractions and explain why decimals are the preferred representation

Suggestions

- "Supermarket Sweep" Activity
  Students will work with partners to determine total costs of items purchased in a store.
- "Holiday Shopping" Activity
  Students will work independently or in small groups to determine what gifts they can purchase (including sales tax and possible shipping/handling charges) for their families.

Resources

- Connected Math 2, Bits and Pieces III
- Crosswalk Coach Grade 6, Domain 1 Lesson 7-8
- Teaching the Common Core Math Standards, The Number System: 6.NS.2

Supplemental Materials and Resources
Literature connection: ***The Doorbell Rang*** by Pat Hutchins ***Full House: An Invitation to Fractions***, by Dayle Ann Dodds ***The Phantom Tollbooth*** by Norton Jester ***Counting on Frank*** by Rod Clement Interdisciplinary connections: Science (volume of rectangular prisms) Social Studies (decimal number to price/cost manipulation)
Tools/Manipulatives
Adding Machine Tape (to create number lines) Calculators Coordinate Grids Decimal Blocks Factor Trees Fraction circles/Fraction pieces Number Lines Pattern Blocks Two Colored Counters

Suggested Formative Assessment Practices/Processes
Teacher created quizzes and exit slips Teacher created exit slips, teacher created quizzes Think-Pair-Share: Students discuss their observations about the relative location of numbers on a number line in relation to their magnitude before sharing their ideas with the class. Agreement Circles: The teacher creates a set of statements (both true and false) related to the topic (example: when plotting a point on a coordinate grid, it does not matter which way you move first). Students stand in a circle and the teacher reads a statement. Give students a few seconds to think. If they agree with the statement, they move to the center of the circle. Fist to Five: Have students self-assess how well they understand the concept with a show of

Differentiation and Accommodations
• Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments