Grade/Subject	Grade 6/ Mathematics
Unit Title	Unit 4: Using Expressions and Equations/Algebraic Reasoning
Overview of Unit	Unit 4 requires students to apply and extend previous understandings of arithmetic expressions to now include using algebraic expressions as well. Students will reason about and solve one-variable equations and inequalities. They will also represent and analyze quantitative relationships between dependent and independent variables. A strong emphasis will be placed on students' ability to move flexibly between real-world problems and algebraic expressions.
Pacing	27 days

Background Information For The Teacher
In 5th grade, students have worked with parentheses, braces, and brackets in numerical expressions. They have learned to evaluate expressions with these symbols. In addition, students have written and interpreted simple numerical expressions, without evaluating them. In this unit, students begin to formalize their understanding of algebra by writing, interpreting and using mathematical expressions and equations. Students begin to make generalizations and use variables in a more sophisticated way. Students represent and analyze patterns and mathematical situations with tables, words, expressions, and equations. Students explore the meaning of variables in familiar formulas such as perimeter, area and volume. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way. They solve simple one-step equations by using number sense, number properties, and the idea of maintaining equality on both sides of an equation. Students know that the solution of an equation is the value of the variable that makes the equation true. Equations apply the rules of arithmetic and algebra to determine equivalence and calculate values. A value can be represented as a number, a numerical expression, an algebraic expression, or an equation. Different symbols in an equation can have the same, or different, values. In mathematical situations and structures, using letters as variables facilitates generalizations and the exploration of numbers and their operations. Students will also use variables to represent two quantities in a real-world problem that change in relationship to one another. They will understand the relationship between an independent and a dependent variable. They will use tables and graphs to investigate these relationships.

The application and extension of previous understanding of arithmetic to algebraic expressions occurs throughout this unit. Teaching students about constants, variables, and expressions is done within the context of real-world situations. All activities used throughout this unit should be related to the real-world in order to help students reason abstractly and quantitatively. A student centered approach will foster students' understanding of the concepts. Through student discourse, and justification of their answers, students should regularly be required to construct viable arguments and critique the reasoning of others in order to further develop their understanding.

The majority of the concepts in this unit are new to the sixth grade curriculum. The level of difficulty in the content and skills introduced in this unit were transplanted from past seventh grade curriculum.

Essential Questions (and Corresponding Big Ideas)
How do we represent the relationship between two quantities? • Relationships between quantities can be represented with multiple forms: language, tables, equations, and graphs. Why is it useful to know different forms of representing relationships between quantities? • Different forms provide different perspectives on the function. • Fluency and flexibility with manipulating mathematical equations allows us to be better, more efficient problem solvers. • Multiple expressions/equations can be written to represent and solve any one given problem. How can real-world situations be represented mathematically? • Mathematical expressions, equations, and inequalities are used to represent real-world situations. • Mathematicians use numbers, letters, and symbols to represent situations in the real world. *How are mathematical expressions, equations, and inequalities used to represent and solve real-world and mathematical problems?* • Numbers, operators, variables, grouping symbols, and other mathematical notation can be used to write and solve an expression or equation that represents a real-world problem. • An unknown mathematical quantity can be represented in an expression as a variable (letter). • An expression may be written to represent a quantity in a different way to make it more compact or to feature different

information.

Core Content Standards	Explanations and Examples
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. This standard concentrates on whole number exponents with a focus on understanding the meaning of exponents and exponential notation such as $3^2 = 3 \times 3$. Students find the value of an expression using exponential notation such as $4^3 = 64$. Students write and evaluate numerical expressions such as: 5 + $2^4 * 6$. What the **teacher** does: • Plan experiences for students to investigate that an exponent is notation representing repeated multiplication such as 10 * 10 * 10 as 10^3. The exponent is the number that tells how many factors of 10 there are. The expression 10^3 is called the exponential expression. Exponential notation was developed to write repeated multiplication more efficiently. • Use patterns to discover why for example 2^0 is one and 35^0 is one. • Allow students to use manipulatives to show $3^2 = 9$ square units. Ask students to model the meaning of $3^3 = 27$ cubic units with manipulatives. • Expand to problems such as 5 + 2^4 and 7 $* 4^3$.	6.EE.1. Examples: • Write the following as a numerical expressions using exponential notation. o The area of a square with a side length of 8 m (Solution: $8^2 m^2$) o The volume of a cube with a side length of 5 ft: (Solution: 5^3 ft^3) o Yu-Lee has a pair of mice. The mice have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) • Evaluate: o 4^3 (Solution: 64) o 5 + 2^4 ● 6 (Solution: 101) 7^2 – 24 ÷ 3 + 25 (Solution: 67) What the **students** do: • Understand the meaning of exponents and exponential notation such as $3^3 = 3 * 3$. • Use appropriate terminology to explain how to evaluate an expression. • Evaluate numerical expressions containing exponents. • Discover that nay base to the zero power is 1. Misconceptions and Common Errors: Some students interpret 3^2 as 3 X 2 = 6. This is a common error. Use a number line representation to model the expression. Also, writing the expanded notation of 3^2 =3 x 3 helps students.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. *a.* Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* *b.* Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* *c.* Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of lengths = 1/2.*	6.EE.2 It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. • r + 21 as "some number plus 21 as well as "r plus 21" • n • 6 as "some number times 6 as well as "n times 6" • s ÷ 6 as "as some number divided by 6" as well as "s divided by 6" Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of the solutions of the four math operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. There are various possibilities for the number they can represent; Students can substitute these possible numbers for the letters in the expression for various different purposes. Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y. There are 4 terms, x^2, 5y, 3x, and 6. There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5 y's or 5 ● y. There is one constant term, 6. The expression shows a sum of all four terms. Examples: • 7 more than 3 times a number (Solution: $3x + 7$)

Parts a-c of Standard 2 emphasizes translating expressions from verbal expressions to numerical ones and from numerical expressions to verbal expressions. Students evaluate expressions given values for the variables such as in the example in part c of this Standard using the order of operations when appropriate. Students identify parts of an algebraic expression including sum, term, product, factor, quotient, coefficients, and constants.

What the **teacher** does:

- Explore with students that letters called variables represent unknown numbers and the same rules that apply in operations with numbers also apply in operations with variables.
- Help students translate verbal expressions into numerical expressions by providing a verbal expression such as "the sum of 6 and 4" and asking them to explore ways to write it with numbers. Encourage students to explore synonyms for operations such as: and, plus, and sum, which can all signify addition.
- Model the notation 6n for 6*n because a number and variable written together means multiply.
- Provide varied practice translating numerical expressions into word form and from a word into variable expressions such as "8 less than 2 times a number is $2x - 8$, "4 times the sum of a number and 2 is $4(x + 2)$," or $2(8 + 7)$ is read as the product of 2 times the quantity or sum of 8+7."
- Help students define parts of an algebraic expression, including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient), as this helps students understand the structure of expressions and explain their process for simplifying expressions. For example: **Terms** are values separated by addition and subtraction such as: $x + 3$ contains 2 terms and $2x - 5$ contains 2 terms.
- Help students concentrate on evaluating algebraic expressions for a given value of a variable, using the order of operations such as the following: Evaluate the expression $2(x + 3^2)$ when x = 5.

6.EE.3. Apply the properties of operations to generate equivalent expressions.
For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent

- 3 times the sum of a number and 5 more (Solution: 3(x + 5))
- 7 less than the product of 2 and a number (Solution: 2x – 7)
- Twice the difference between a number and 5 (Solution: 2(z – 5))
- Evaluate $5(n + 3) - 7n$, when $n = \frac{1}{2}$
- The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where *c* is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.
- The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

What the **students** do:

- Recognize that variables represent unknown quantities.
- Translate verbal expressions and numerical expressions into verbal expressions.
- Communicate orally and/or in writing about translating and evaluating variable expressions using precise mathematical language, including variables, coefficients, constant, and term.
- Evaluate expressions for given values of variables using the order of operations when appropriate.

Misconceptions and Common Errors:
Some students misunderstand or incorrectly read expressions. Students often confuse x^3 with 3x. To address this situation, ask students to create a chart with the meaning of x^3 and 3x such as:

x^3 means	3x means
X times x times x X to the third power	3 times x x + x + x

expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression
6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

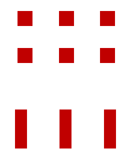
Standard 3 spotlights applying the properties (distributive property, the multiplicative identity property of 1, and the commutative property for multiplication of operations) with expressions involving variables to generate equivalent expressions.

<u>What the **teacher** does:</u>

- Provide learning opportunities for students to use multiplication to interpret 3(2+x) as three groups of (2+x). Have students create an array with three columns and x + 2 in each column to show the meaning of 3(2+x). Ask students to discuss and explain why 3(2+x) is equal to 6+3x.
- Use manipulatives to interpret y as referring to one y, and y plus y plus y is 3y. Discuss the distributive property, the multiplicative identify property of 1, and the commutative property for multiplication to prove that
  y + y + y = 3y.
- Encourage students to generate equivalent expressions for 5(x-2) and 4x + 3y.
- Ensure students have opportunities to talk with the teacher and others to make sense of equivalent expressions.

6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).
For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

6.EE.3. Students use their understanding of multiplication to interpret *3 (2 + x). For example, 3 groups of (2 + x).* They use a model to represent x, and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x.*
An array with 3 columns and *x* + 2 in each column:



Students interpret *y* as referring to one *y*. Thus, they can reason that one *y* plus one *y* plus one *y* ***must be*** *3y.* They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that *y* + *y* + *y* = 3*y*:
y + *y* + *y* = y x 1 + *y* x 1 + *y* x 1 = y x (1 + 1 + 1) = *y* x 3 = 3*y*

<u>What the **students** do:</u>

- Understand that the properties used with numbers also apply to expressions with variables.
- Apply the properties of operations with expressions involving variables to generate equivalent expressions.

<u>Misconceptions and Common Errors:</u>
When using the distributive property, some students may multiply the first term in the parentheses but forget to do the same to the second term. To address this error, give students a plastic ziplock bag of approximately 25 counters in two different colors mixed in each bag. Direct students to empty the bag and count the number of each color such as there are 14 yellows and 11 reds. Ask students to use the distributive property to write an expression to show how many of each color would be in 4 bags. Students write the expression 4(14Y +11r). Using the distributive property, the expression is 56Y + 44r. Interpret this as 56 yellow and 44 reds in 4 bags. Provide other examples.

Give students error analysis problems such as the following: "Fred said 3(2+x) and 6+x are equivalent expressions. He was incorrect. Tell Fred what he did incorrectly." One solution is to remind Fred that 3 must be distributed through both terms in parentheses.

This standard focuses on combining like terms in expressions. Students will substitute values into expressions to prove equivalence. For example, Are 3(x+4) = 3x + 12 equivalent expressions? Substitute a numerical value for x such as 2. Then, 3(2+4) =18 and (3x2) +12 = 18 so the expressions are equivalent.

What the **teacher** does:

- Have students explore adding or subtracting like terms as quantities that contain the same variables and exponents. For example, 5x+4x are like terms and can be combined as 9x; however, 5x + $4x^2$ are not like terms since x and x^2 are not the same. Manipulatives such as algebra blocks or tiles can be used to explore this concept.
- Provide practice for students to prove equivalence with substitution. They will use substitution to verify that both expressions are equivalent such as 3(2+x) = 6+ x. They should substitute any number for x. If he expressions have different values, they are not equivalent.
- Ensure students have adequate practice with equivalent expressions using the associative, commutative, and distributive properties.

Using manipulatives such as algebra blocks or tiles is also helpful in modeling the distributive property.

6.EE.4**.** Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example:

- Are the expressions equivalent? How do you know?

$4m + 8$ $\quad$ $4(m+2)$ $\quad$ $3m + 8 + m$ $\quad$ $2 + 2m + m + 6 + m$

What the **students** do:

- Explore the concept of like terms and apply combining like terms in expressions accurately.
- Reason that two expressions are equivalent through the use of substitution.
- Explain reasoning to other classmates and the teacher using precise mathematical vocabulary.

Misconceptions and Common Errors:

Some sixth graders do not recognize when letters are used to represent variables and when letters are used to represent units of measure such as 4m and 4 m as in meters or 3h and 3 h as in hours. Use contextual examples to distinguish between the two.

Some students may continue to combine 4x and $4x^2$. Use a manipulative such as square tiles to demonstrate the difference between the two terms.

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.5. Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of

The center of attention for this standard is solving an equation or inequality as a process of answering the following question: Which values from a specified set make the equation or inequality true? Students simplify numerical expressions by substituting values for given variables and use substitution to determine whether a given number in a specified set makes an equation true or which set of numbers makes an inequality true. Limit solving inequalities to selecting values from a given set that would make the inequality true. For example, find the value(s) of y that will make $7.2 + y \geq 9$. Select your value(s) from the set = $\{1, 1.3, 1.8, 2, 3\}$.

<u>What the **teacher** does:</u>

- Provide experiences for students to focus on understanding the meaning of solving an equation before developing the procedural knowledge. Start with a balance scale model to represent and solve equations. Say, "There are 42 centimeter cubes on the left side of he scale and 200 cubes on the right side of the scale. All the blocks are the same size. How many cubes must be added to the left side of the scale to make the scale balance?"
- Pose questions for variable equations such as the following: What numbers could possibly be the solution for x + 17 = 27? Note that

students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities. Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation 26 + *n* = 100 where *n* is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100". Students ask themselves "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem.

o Reasoning: 26 + 70 is 96. 96 + 4 is 100, so the number added to 26 to get 100 is 74.

o Use knowledge of fact families to write related equations: *n* + 26 = 100, 100 - *n* = 26, 100 - 26 = *n*.

o Select the equation that helps you find *n* easily.

o Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of *n*

o Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.

o Bar Model: Each bar represents one of the values. Students use this visual representation to
demonstrate that 26 and the unknown value together make 100.

100	
26	***n***

Examples:

- The equation 0.44*s* = 11 where *s* represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to equations should not require using the rules for operations with negative numbers.

- Supply scenarios where the solution is a single answer or multiple answers for the students to explore. This helps students establish the difference between equations and inequalities.
- Ask questions for variable inequalities such as the following: What numbers could possibly be the solution for x + 17 > 27? Present possible solutions for students to select from. Include rational numbers in the set.
- Emphasize simple equations for students to solve using reasoning and prior knowledge such as "Maria has 42 dollars in her bank. For her birthday she received some more dollars and now has $200. How many dollars did she receive for her birthday?" Provide a set of possible solutions so that students may use substitution to find the solution. One possible set is $\{78, 58, 158, 258\}$.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

This standard concentrates on writing expressions using variables that represent real-world or mathematical problems. Students learn that a variable represents an unknown number or any number in a specified set.

determine your answer. Show that your solution is correct using substitution.

- Twelve is less than 3 times another number can be shown by the inequality 12 < 3*n*. What numbers could possibly make this a true statement?

What the **students** do:

- Use precise mathematical vocabulary to explain the differences between equations and inequalities.
- Discover that solutions to inequalities represent a range of possible values rather than a single solution.
- Reason the value(s) that make an equation or inequality true and select from a given set of values.
- Simplify numerical expressions by substituting values for given variables.

Misconceptions and Common Errors:

Many students have difficulty understanding that an inequality can have more than one solution. The best way to work on this concept is to use real-world examples that are familiar to students. For example, I have $25 and want to buy some bracelets. The bracelets cost $8 each. How many could I buy? This results in the inequality $8b \leq 25$ where b is the number of bracelets I can buy. Since students are not solving inequalities in this standard, if you include a negative number in the set of possible solutions, have a discussion about how the negative value only works for the equation and not the real-world scenario.

6.EE.6**.** Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. (Solution: 2*c* + 3 where *c* represents the number of crayons that Elizabeth has.)
- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: 28 +

What the **teacher** does: • Provide a variety of experiences for students to write expressions for solving real-world word problems. Focus on reading algebraic expressions to make the connection that a variable represents a number such as the following: Sean has five more than twice as many pencils as John. Writ an algebraic expression to represent the number of pencils Sean has. 2j + 5 where j represents the number of pencils John has. • Encourage students to identify what the variable represents in each real-world scenario when they write an expression. • Help students describe problem situations solved using an equation such as 4c + 5 = 25, where c represents the cost of an item. Provide other problems where a variable represents any umber in specified set.	0.35*t* where *t* represents the number of tickets purchased) • Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned. (Solution: 15*h* + 20 = 85 where *h* is the number of hours worked) • Describe a problem situation that can be solved using the equation 2*c* + 3 = 15; where *c* represents the cost of an item. • Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $5.00 + *n)* • The commutative property can be represented by *a* + *b* = *b* + *a* where *a* and *b* can be any rational number. What the **students** do: • Understand that a variable represents a number or a specified set of numbers. • Represent real-world scenarios with variable expressions, identifying what the variable represents. Misconceptions and Common Errors: Some students continually misrepresent real-world scenarios with expressions. They each make different errors. Do an error analysis on the work of the students who repeatedly make errors. Are they mistaking what the variable is? Do they have trouble translating verbal expression to variable expressions? Are they seeking to write equations instead of expressions? Write error analysis questions for the students to solve that use each of common student errors being made in class that you have identified. An example of such a problem is as follows: Fred wrote x+6 when asked to find an expression for Sam has 6 times as many fish as Paul. Fred was incorrect. Write Fred a note explaining his error. Note: Do not use real students names. This meant to clarify misconceptions generally, not embarrass students who made the errors initially.
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p*, *q* and *x* are all nonnegative rational numbers.	6.EE.7. Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies. Example:

Attention for Standard 7 is placed with solving equations for real-world and mathematical problems that involve positive rational numbers and zero. To solve the equation, students can draw pictures such as this example: Juan spent \$48.99 on three T-shirts. If each shirt is the same amount, write an algebraic equation that represents this situation and solve to determine how much one T-shirt costs. The picture created is a bar model chart. Each bar is labeled S for T-shirt, so each pair of jeans costs the same amount of money. The bar model represents the equation 3S = \$48.99. To solve the problem, students divide the total cost of \$48.99 by 3.

\$48.99		
S	S	S

What the **teacher** does:

- Pose problems for students to explore solving equations based on real-world scenarios such as, Corry bought 6 CD's that each cost the same amount. Without tax, he spent \$89.94. How much did he spend on each CD? Write and solve an equation to solve the problem. Note that problems should only use positive rational numbers (including 0), fractions, and decimals. Encourage students to illustrate the equation in problem situations by drawing a picture or using reasoning and prior knowledge. Solving equations using reasoning, pictures, diagrams, and prior knowledge allows students to develop effective strategies on their own.
- Ask students to generate equations based on situations from their daily lives such as texting friends. Have students explain the meaning of the variables used.
- Ensure students have opportunities to talk with the teacher and each other to make sense of equations in the form of x + p = q and px = q.

6.EE.8. Write an inequality of the form x>c or x<c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the

- Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

\$56.58		
J	J	J

Sample Solution: Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled *J* is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3*J* = \$56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10 x 3 is only 30 but less than \$20 each because 20 x 3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That's \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 (15+3+0.86). I double check that the jeans cost \$18.86 each because \$18.86 x 3 is \$56.58."

- Julio gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julio has left.

(Solution: 20 = 1.99 + 6.50 + *x*, *x* = \$11.51)

20		
1.99	6.50	money left over (m)

What the **students** do:

- Solve equations that represent real-world mathematical problems that involve positive rational numbers and zeros.
- Model real-world situations with equations and use a variety of strategies to solve them.
- Use precise mathematical vocabulary to communicate with the teacher and classmates.

Misconceptions and Common Errors:

Some students may need additional, on-going practice with writing and solving equations. Use advertisements in newspapers to generate real-world scenarios that may be used to write and solve the equations.

form x >c or x<c have infinity many solutions; represent solutions of such inequalities on number line diagrams.

The essence of Standard 8 is graphing inequalities on a number line and writing inequalities to solve real-world mathematical problems. Students check by substitution to determine if the graph of an inequality is correct.

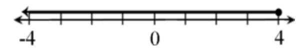
<u>What the **teacher** does:</u>

- Provide opportunities for students to represent inequalities on a number line. Present problems such as, less than $200.00 was spend by Mrs. Smith for the class party. Write an inequality to represent this amount and graph this inequality on a number line. Explain that the open circle above the 200 means that 200 is not included in the solution boundary set. The ray represents all number in the solution set.
  Check the solution by having students each select a number represented on the number line as part of the solution set and determine if it makes the inequality x < 200 true. Ask students if -200, which makes the statement x < 200 true, is a realistic answer to the word problem. Then, facilitate a class discussion.

6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

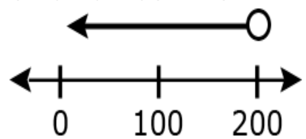
6.EE.8.Examples:

- Graph $x \leq 4$.



- Jonas spent more than $50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.
- Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality

on a number line.



Solution*:* 200 > *x*

<u>What the **students** do:</u>

- Discover that a variable can stand for an infinite number of solutions when used in inequalities.
- Graph inequalities on a number line.
- Write inequalities to solve real-world mathematical problems.
- Check by substitution to determine if the graph of an inequality is correct.

<u>Misconceptions and Common Errors:</u>
Some students may need additional on-going practice with writing inequalities to represent real-world mathematical situations. Use advertisements in newspapers to generate ideas of real-world scenarios that can be used to write inequalities to represent an amount. Ask students to talk about the problems and the number lines they created to show the inequalities.

6.EE.9**.** Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.
Examples:

This standard accents using variables to represent two quantities in real-world scenarios. Students recognize that a change in the independent variable creates a change in the dependent variable, such as the following: As x changes, y also changes. Emphasis is placed on writing an equation to express the quantity in terms of the dependent and independent variables. Students also identify relationships between tables, graphs, and equations and relate these back to the equations.

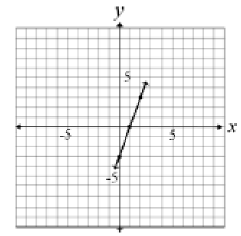
What the **teacher** does:

- Provide experiences for students to understand multiple representations such as tables, equations and graphs that can be used to analyze relationships between quantities. Students should describe the relationships using language. Ensure students understand that each representation shows the same relationship.
- Include numerous situations for students to analyze and determine the unknown that is dependent on the other components such as how far someone travels is dependent on the time and rate.

• What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

• Use the graph below to describe the change in *y* as *x* increases by 1.



• Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account.

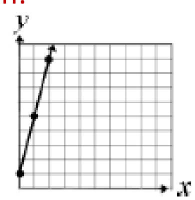
o Language: Susan has $1 in her savings account. She is going to save $4 each week.

o Equation: $y = 4x + 1$

o Table:

x	y
0	1
1	5
2	9

o Graph:



What the **students** do:

- Use variables to represent to quantities.
- Identify relationships between tables, graphs, and equations.

	• Recognize that a change in the independent variable creates a change in the dependent variable such as the following: As x changes, y also changes. • Write an equation to express the quantity in terms of the dependent and independent variables. Misconceptions and Common Errors: Some students may confuse what a graph represents. To help, have students explain in their own words what the graph means.
Standards for Mathematical Practice	**Explanations and Examples**
Apply and extend previous understandings of arithmetic to algebraic expressions. **6.EE.1, 6.EE.2, 6.EE.3, 6.EE.4** The focus of this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as $3^3 = 27$, using the appropriate terminology to explain how to evaluate an expression. Students are applying properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.	
MP2. Reason abstractly and quantitatively.	Sixth graders decontextualize to manipulate symbolic representations by applying properties of operations.
MP4. Model with mathematics.	Students model real-world scenarios with equations and expressions.
MP6. Attend to precision.	Students communicate precisely with others and use clear mathematical language when describing expressions.
Reason about and solve one-variable equations and inequalities. **6.EE.5, 6.EE.6, 6.EE.7, 6.EE.8** Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities of the form x>c or x<c and use number line representation to show the solutions of inequalities.	
MP1. Make sense of problems and persevere in solving them.	Sixth graders solve real-world and mathematical problems through the application of algebraic concepts. They look for meaning of a problem and find efficient ways to represent and solve it.
MP2. Reason abstractly and quantitatively.	Grade 6 students use properties of operations to generate equivalent expressions and use the umber line to understand

MP4. Model with mathematics. **MP6. Attend to precision.** **MP7. Look for and make use of structure.**	multiplication and division of rational numbers. Students write expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. They use number lines to compare numbers and represent inequalities. Students communicate precisely with others and use clear mathematical language when describing equations and inequalities. Sixth graders apply properties to generate equivalent expressions and solve equations by the subtraction property of equality.
Represent and analyze quantitative relationships between dependent and independent variables **6.EE.9** The focus for this cluster is using variables to represent two quantities in a real-world problem that change in relationship to one another. Students write an equation and analyze the relationship between the dependent and independent variables using graphs and tables. **MP1. Make sense of problems and persevere in solving them.**	
MP4. Model with mathematics. **MP6. Attend to precision.** **MP7. Look for and make use of structure.**	Sixth graders solve real-world problems through the application of algebraic concepts. Students model real-life situations with mathematics and use variables to represent two quantities in real-world problems. Problem situations are modeled symbolically, graphically, tabularly, and contextually. Students communicate precisely with others and use clear mathematical language when describing dependent and independent variables. Sixth graders represent mathematics to describe mathematics to describe a situation with either an equation or diagram and interpret the results.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
• Parts of an expression: variables, coefficient, constants • Order of Operations is the conventional order when solving a multi-step arithmetic problem • Properties of Operations: distributive property, commutative property, associative property • Exponents are used to show repeated multiplication • Evaluating an algebraic expression describes finding a numeric value for the expression. • An unknown mathematical quantity can be represented in an expression as a variable (letter). • Purpose and use of variables • Two quantities change in relationship to one another (independent and dependent variable) • The difference between an expression and equation. • Graph • Table	• WRITE expressions and equations • READ expressions • EVALUATE expressions • SIMPLIFY expressions • VIEW one or more parts of an expression as a single entity. • USE mathematical terms appropriately • APPLY properties of operations • USE order of operations • GENERATE equivalent expressions • IDENTIFY two equivalent expressions • TRANSLATE real-world problems into expressions • SOLVE mathematical problems • IDENTIFY parts of an expression • RECORD operations with letters and numbers • USE variables • REPRESENT dependent and independent variables • ANALYZE relationships • USE graphs and tables • REPRESENT numbers • UNDERSTAND variable representation • SOLVE problems by writing and solving equations of the form $x + p = q$ and $px = q$ (p,q and $x \geq 0$) • SOLVE equations
UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*	
Students will understand that:	

- Mathematical expressions, equations, and inequalities are used to represent and solve real-world and mathematical problems.
- Relationships between quantities can be represented with multiple forms: language, tables, equations, graphs.
- Flexibility in manipulating expressions to suit a particular purpose helps with solving problems efficiently.
- An expression may be written to represent a quantity in a different way to make it more compact or to feature different information.
- Symbols/letters allow us to solve mathematical problems when we don't have all the information.

Common Student Misconceptions for this Unit

Students might:

- Think that the order of operations always goes from left to right.
- Think that to simplify an exponent you multiply the base times the exponent.
- Think that in expressions such as 4x where x=2, that 4x is equivalent to 42 rather than 4 times 2.
- Think that when reading expressions constants and variables always remain in the same order. (Example: 4 less than a number is 4-n instead of n-4.)
- Only multiply the number on the outside of the parentheses by the first number inside the parentheses when using the distributive property. (Example: 4(n-7) Students might write 4n-7 instead of 4n-28.
- Think the independent variable is always first.
- Think the independent and dependent variables are interchangeable.
- Forget to use the inverse operations to isolate the variable when solving equations.
- Confuse the terms expression and equation.
- Look at the pattern for one variable rather than the relationship between two variables. (They might tell what happens to x to get the next x value rather than describing what happens to x to get y.)

Unit Assessment/Performance Task	DOK

• The Gym • Unit 4 Test Part A and Part B	

Vocabulary
• Base • Constant • Coefficient • Dependent Variable • Distributive Property, Commutative Property, Associative Property • Equation • Equivalent • Evaluate • Exponents • Exponential Notation • Expression/Algebraic Expression • Factor • Formula • Function • Graph • Independent Variable • Inequality • Like Term • Numerical Expression • Rational Numbers

- Simplify
- Substitution
- Sum, Difference, Product, Quotient
- Table
- Term
- Variable

Key Learning Activities/Possible Lesson Focuses (order may vary)

The following activities are broken into "lessons", even though each may take more or less than one class period depending on school schedule.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

<u>Reading and Writing Expressions</u>: ****Note for teacher: This is a review from Unit 1: Expressions and Equations. Students should already have an understanding of reading and writing expressions.*

Provide students with stacks of cards to match up: half with expressions, half with written words (as you have below). Once all cards are matched up, students choose one for which to write a real world situation (word problem).

Five plus some number = $5 + x$

Twelve less a number = Twelve minus a number = $12 - x$

Twelve less than a number = A number minus twelve = $x – 12$

Be sure that phrases and expressions build in complexity to include all four operations and, ultimately, expressions with multiple different operations. Four more than three of a number = $4+(3n)$

For practice, continue translating real-world situations as word phrases and algebraic expressions. Collect all examples and review. Choose a few to use to teach evaluating expressions in the next lesson.

Evaluating Expressions

Complete a Think-Pair-Share with students about the Order of Operations (about 30-45 seconds) in order to review PEMDAS. Review common misconceptions: exponents from previous lesson as well as new notation for multiplication and division operations (when a number and a variable are directly next to one another, it means to multiply, using a raised dot (or asterisk) between two numbers means to multiply, and separating two numbers with a fraction bar means to divide. Identify parts of an expression, then begin to use substitution to evaluate. Use the previous lesson's problems and begin to evaluate expressions at specific values of their variables.

$(5 + x;\ x = 7)$ $(4 + (3n);\ n = 6)$ $(5 + y^2 - 4; y = 3)$

Using Variables and Writing Expressions when Solving Real-world Problems

Write the following on board: "Tom earned $20.00 shoveling driveways on Saturday. He earned some more money on Sunday." Ask students, "How can we write an algebraic expression that shows how much money he has earned?" ($20.00 + m) Discuss how the variable represents an unknown number.

Lisa has a summer job selling magazines. She earns $9.00 per hour. Write an expression to represent the amount of money she earns. ($9.00h)

Fred has 50 more than twice as many baseball cards as Kenny. Write an expression to represent the number of baseball cards Fred has. (2c + 50)

Equivalent Expressions

As a whole group, ask students to solve 4 x 21 in their head. Let students share their responses, and lead into discussing the distributive property (4 x 21 = 4 x (20+1) = (4 x 20) + (4 x 1)). Relate this to previous lessons on expressions by including real-world examples requiring the use of variables. (i.e. perimeter of a rectangle can be found by adding all 4 sides together, or by adding

the length and width together, then multiplying by 2, or by multiplying the length by 2, multiplying the width by 2, then adding those two products together: $l + w + l + w = 2(l+w) = 2l + 2w$)
Ask students, in pairs or small groups, to come up with as many ways as they can to get the number 10 (i.e.: 3+7, 12 – 2, 2 x 5, or 30/3). Have students write 3 or 4 of their examples as equivalent expressions, using equals signs: 3+7 = 12 – 2 = 2 x 5 = 30/3. If examples, such as 1 + 2 + 3 + 4 do not already arise, be sure to lead students in this direction. Ask students which is easiest to read and understand: 2 x 5, 1 + 2 + 3 + 4, or 10? Why? Use this to discuss combining like terms. Provide students with a variety of expressions. Have them determine which ones are equivalent. Students should be reminded to choose a value for their variable(s) to check whether or not they are correct. (i.e.: Students may think $4m + 8 = 4(m+8)$. However, if they choose a value for m, and follow the order of operations, they will find that they are not equal, and $4m + 8 = 4(m+2)$.
When possible, and if needed, allow students to use manipulatives (possibly with a scale/balance) to deepen their understanding of equivalence.

Independent vs. Dependent Variables:
Given a set of index cards with independent and dependent variables written on them (color-coded by matches), students will decide which is the independent and dependent variable. Students can work in groups of 4. Each group would be given 4 sets of cards (8 cards total). 2 cards of the same color would go together, one being the independent, and the other being the dependent variable. Students would decide which one affects the other one. From their 4 sets, they must write an explanation for one of them (as a group), telling how one would affect the other. (Example: One card might say: total cost of purchase and the other one might say: number of songs purchased on iTunes. Students would discuss and determine that the number of songs purchased affects the total cost of the purchase. The relationship is that as you buy more songs, the cost goes up). Once all groups are finished, all groups in the classroom will work together to see if they can find any other relationships within the classroom. (For example, in the

iTunes songs/total purchase cost scenario, the number of songs was the independent variable, and the cost was the dependent variable, but another group in the class might have an independent variable that says "gift card balance" and a dependent variable that says "number of songs I can purchase on iTunes".) As a challenge, the teacher might choose not to color code the cards, and leave matches up to the students' interpretation, given that they have reasonable explanations. At the end of the activity, have students write all of the matches in a chart in their notebook (independent/dependent variable).

Using Tables to Illustrate the Relationship Between Variables

Have kids work on "What's My Rule?" (An input/output table where all of the inputs are given and a few of the outputs are given. Students must determine the relationship between the inputs and outputs in order to find the remaining missing outputs.) The problems should start off easier, and build in complexity. Once all of the tables are completed, students will look at the independent/dependent variable narrative index cards from the previous lesson to see if they can find any scenarios where the values seem feasible for a situation, discussing why or why not they could go together.

The following web-site can be used to practice determine the rule for a function:
http://www.mathplayground.com/functionmachine.html

Next, give students a narrative and have them complete a table and write an expression to match the narrative. (Example: You are going jet skiing for the day. It costs $45.00 an hour to rent a jet ski. Fill in the table below to determine how much it would cost to rent a jet ski for 1, 2, 3, 4, or 5 hours. Use your table to write an expression that someone could use to determine the cost of renting a jet ski for any number of hours.)

Then, give students another narrative (or a few others), similar to the example above about renting a jet ski, where the table is not given, and students must create it themselves.

Using Graphs to Illustrate the Relationship Between Variables

Give students a variety of graphs. For each graph they must describe what happens to y as x increases by one. (See 6.EE.9 Explanations and Examples for a visual). Graphs may build in complexity to include graphs where students must see what happens to y as x increases by a different number in order to determine what happens as x increases by one because it is not clearly displayed on the graph. Once students describe the relationship in words, they will use their descriptions to write equations. Next, have students pick a graph for which they can determine labels for the independent and dependent variable because they feel they could be related in the way the graph illustrates.

Suggested Resources:

Using Tables, Graphs, and Expressions to Illustrate the Relationship Between Variables

Given a real-world situation, students must create a table, create a graph, describe the relationship between the variables, and write an expression to match the scenario. Students should be able to complete these in any order they see fit. Students could complete this independently, with a partner, or in a group of 3.

Next, given a real-world scenario, students can choose one method of illustrating the relationship between the variables: table, graph, or expression. Once they've used one method, they must then explain the relationship between the variables.

Suggested Resources:

Solving One-Step Equations: Solving equations was taught in Unit 1. The following activities are intended as refreshers and reinforcement.

Ask students to think back to Unit 1 (teacher may provide some examples of problems students worked on during this unit) and give advice to future students learning to solve algebraic equations: common errors and tips.

Provide students with more real-world scenarios for which they must write an equation. Students are then to solve the equations that they have written, and then give the answer in terms of the real-world problem.

Solving Addition Equations: To solve an addition equation, subtract the same number from each side so that the variable is by itself on one side. Solving an addition equation can be modeled using algebra tiles and a balance scale. After solving an equation, students should always check your solution.

Solving Subtraction Equations: To solve a subtraction equation, add the same number to each side so that the variable is by itself on one side. You add the same number to each side of the equation to undo the subtraction and keep both sides of the equation equal to each other. After solving an equation, you should always check your solution.

Solving Multiplication and Division Equations: To solve a multiplication or division equation, try to get the variable by itself on one side of the equation. You can use multiplication and division to undo each other. To solve a multiplication equation, divide each side by the number that the variable is multiplied by. To solve a division equation, multiply each side by the divisor.

Solving Real-world Problems Using Equations

Provide students with a variety of word problems and require them to write and solve an equation for each. (i.e. Doug had $53.00. He earned some money mowing lawns. Doug now has $70.18. How much money did Doug earn? $53 + x = 70.18$, $x = 17.18$, so Doug earned $17.18 mowing lawns. Require students to explain how they arrived at their answers. For example, students may

use a bar model or inverse operations. Next, provide students with a variety of equations, and require them to come up with their own real world problems to match the equations. (i.e. 32n = 96 Students may write a word problem that says, April has some boxes of crayons. Each box holds 32 crayons. April has 96 crayons total. How many boxes of crayons does April have?)
Suggested Resources:

-NCTM-Building Bridges (http://illuminations.nctm.org/LessonDetail.aspx?id=L247)

Equations and Inequalities

Introduce the idea of an equation as a balance, with the equals sign as the balancing point. Depending on availability your class can use the Pan Balance exercise from the NCTM Illuminations site (http://illuminations.nctm.org/ActivityDetail.aspx?ID=10). In lieu of computers, use actual balances, or one balance for a class demonstration and drawings of balances. Demonstrate how the value of an unknown quantity can be determined by isolating the variable through inverse operations. For example, $4 + x = 9$ can be modeled with a balance where on one pan of the balance there are four visible objects and an unknown quantity (in a paper bag or something) and the other pan has 9 of the same objects. Remove the four objects from the pan and allow students to observe how the balance is lost. Demonstrate or guide the students to the solution of removing four objects from the other pan will return balance to the pans. Now the unknown number of objects in the first pan must be equal to the number of objects left in the second pan. The following notation is not intended for the students, but only to clarify the math for the instructor.

$4 + x = 9$; If $(4 + x) – 4 = x$ and this balances with $9 – 4 = 5$, then x must be 5.

Inequalities (as an extension for enrichment)

Build on the previous lesson. Begin with a pan set up of $4 + x \neq 9$. Have more than five objects in the first balance unknown. A balance that is upset means that the pans are not equal. The set up represents an inequality. Ask the students how many objects could be invisible that make the first

balance heavier. Discuss. Set up another situation where the left balance is light. Discuss. Guide the students to the follow representations $4 + x > 9$ for the first example and $4 + x < 9$ for the second one. Introduce the use of number lines to indicate the infinite possibilities that will make the expression an inequality.
Supplemental Materials and Resources
NCTM Pan Balance http://illuminations.nctm.org/ActivityDetail.aspx?ID=26 Brain Pop www.brainpop.com/math/algebra/equationswithvariables/ Algebraic Expressions Millionaire http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html The Futures Channel http://www.thefutureschannel.com Math Play.com http://www.math-play.com/Algebra-Math-Games.html Weight the Wangdoodles http://www.mathplayground.com/wangdoodles.html **Literature connections:** Two of Everything by Lily Toy Hong One Grain of Rice by Demi The King's Chessboard by David Birch and Devis Grebu Bats on Parade by Kathi Appelt Bat Jamboree by Kathi Appelt **Interdisciplinary connections:** -Science: Using formulas for speed, velocity, volume, density, acceleration, etc.
Tools/Manipulatives

Algebra Tiles and/or square tiles (paper or commercially produced) Calculators Scale/Balance Paper strips for tape Diagrams/bar models Number Line Cubes such as linking cubes, Unifix Cubes ™, Wooden Cubes

Suggested Formative Assessment Practices/Processes

- Exit Card: For the lesson on independe
  with a scenario written on it. They mus
  variable and explain the relationship be
- Card Sort – Students collaboratively so
  independent/dependent variable or ma
- Create a problem – Given an equation/
  match it.
- Teacher observation/anecdotal notes u
  misconceptions during a lesson or at th
- Problem of the Day
- Lesson Quizzes
- Entrance Slips

Differentiation and Accommodations

- Provide graphic organizers
- Provide additional examples and opportunities for repetition
- Provide tutoring opportunities
- Provide retesting opportunities after remediation (up to teacher and district discretion)
- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments