Grade/Subject	Grade 7A/Accelerated Mathematics
Unit Title	Unit 3: Proportional Relationships and Percentages
Overview of Unit	In this unit, students will apply and extend previous understandings of ratios, unit rates, proportional reasoning and percentages begun in the 6th grade. Students are now expected to be able to compute unit rates with non-whole numbers. Students previously only had to be able to formulate real world ratios given two quantities; now greater depth is required with students expressing proportional relationships within coordinate plane, recognizing the constant, formulating equations representing the situation and determining the graphical representation of singular unit rate. In addition, students need to use previous knowledge of setting up percentage proportions to find real world multistep solutions for taxes, interest, commissions, percentage increase/decrease, scale drawings, etc.
Pacing	Grade 7 Accelerated Mathematics: 9-11 days

Background Information For The Teacher
In sixth grade, students begin to formalize ratio as a particular kind of comparison between two quantities. A ratio is a comparison between two values of those quantities (numbers) that does not change if they are *varied multiplicatively by the same factor*. What is fundamentally different and new for students is the idea that the amount of two quantities can be in the same relationship (and hence be referred to as "equal") even though the amount of each of the two quantities gets larger or smaller. The understanding that equal means "preserving the particular kind of relationship" between the values is an important cognitive achievement for students in the seventh grade. A constant of proportionality in a particular context describes the ratio between two quantities is essential for students to keep track of the order of the proportional quantities. Students learn to be flexible, adaptable, precise, and sensitive to context in their problem solving. They express the constant of proportionality in either way as appropriate to the context. Geometric uses in using proportions are introduced in an additional real world situation in the use scales and creating maps. The most complex forms of problems with ratios and percents involve those with multiple steps. A fairly common one is when students must figure out "percent increase" or "percent decrease." Similar to the problematic with "more than" a percentage less than 100, English and mathematics spar and it is important for students to consider the apparent contradictions in language and learn the different ways that, as a society, we conventionally describe percent increase and decrease.

Essential Questions (and Corresponding Big Ideas)
Where do we use proportions and ratios in everyday life? ● Whenever we compare two quantities there is a proportional relationship. How can I model unit rate and proportional relationships? ● Using the coordinate plane we can determine the unit rate and the proportional relationship of two quantities. How can I use what I already know about percentages and proportions to find solutions to real world situations such as tax, commissions, percent increase/decrease or interest? ● The method for finding the solution is the same as previous methods of setting up a proportion and finding the missing value; you first must identify the components of the proportion setting the part/whole = the percent/100. How can I use algebra to find the solution to proportional situations? ● Total = the constant times the unknown quantity How can the use of proportions help in finding measurements in geometric situations? ● Set up the proportion by comparing two given quantities and then similarly equating it to another ratio where there is an unknown amount.

Core Content Standards	Explanations and Examples
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: "Directions to make a table cloth call for ¾ yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 yards, how much ribbon will be needed?" The proportion is $\frac{\frac{3}{4}}{\frac{2}{1}} = \frac{x}{3}$. To cross multiply: $3*\frac{3}{4} = 2x$ Problems for this standard should be multi-step and include contexts with simple interest, tax, tips commissions, percent error, percent	7.RP.3 Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value. Examples: ● Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015? ○ A student might say: "The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40." $4.17 + 4.17 + (0.24 • 4.17) = 2.24 x 4.17 100% | 100% | 24% $4.17 | $4.17 | ?

increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices.

To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case, 10-2=8. Take the difference 8, and divided by the original number 8/2 = 4. Multiply the quotient by 100: 4 x 100 = 400%.

- A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

$37.50 Original Price of Sweater	
33% of $37.50	67% of $37.50

  - The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.
- A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Discount 40% of original price	Sale Price - $12 60% of original price
Original Price (p)	

0.60p = 12

- At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.
- A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal? After eating at a restaurant, your bill before tax is $52.60 The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill. The amount paid = 0.20 x $52.50 + 0.08 x $52.50 = 0.28 x $52.50

What the **teacher** does:

- Focus time on the vocabulary for this standard. Paper foldables, word walls, graphic organizers, using words in context, and writing stories all give students a chance to clarify the meaning of these terms, which they may encounter in daily life but not fully understand. Bring in items familiar to students such as tennis shoes a six-pack of soda, and so on and use them to model situations that use the vocabulary. Vocabulary should include simple interest, tax, tip/gratuity, discount, commission, fees, sale, markup, markdown, and original price.
- Use cross-multiplication to solve problems involving proportional relationships. Use numbers in your problems that do not lend themselves easily to mental arithmetic.

What the **students** do:

- Explore use of the vocabulary words in this standard by finding examples in the media and explain how they are used in each situation.
- Solve problems involving proportions using cross-multiplication.
- Solve problems involving percent error and percent increase/decrease.
- Use the structure of percent error and percent increase/decrease problems to explain how the formulas for these concepts are similar.

Misconceptions and Common Errors:

Students may have misconceptions about the vocabulary commonly used in the media such as sale, discount, and tax. It is important to discuss what students already know about these words in order to correct any pre-existing misconceptions. For individuals with difficulties with particular words, use graphic organizers such as the Frayer model. Acting out situations can help students remember certain steps. For example, acting out shopping for a pair of

● Begin with single-step problems and move to multi-step using a wide variety of contexts. Make use of everyday examples such as finding sales on line, in print media, and on TV. ● Ask students to write problems that can be solved with setting up proportions prompted by media ads. ● Introduce students to percent increase/decrease and percent error problems. Encourage students, through questioning, to discover the similarities among the formulas for these concepts.	tennis shoes and a tennis racket and paying tax at the register will help students remember that tax is calculated on the cost of the total bill where the items bought need to be added up before tax is calculated.
Standards for Mathematical Practice	**Explanations and Examples**
Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1, 7.RP.2, 7.RP.3 These standards extend what students learn in Grade 6 about ratios to analyzing proportions and proportional relationships. Students calculate unit rates with complex fractions and move to recognizing and representing proportional relationships in equations and on graphs. These skills and understandings are used to solve multi-step ration and percent problems involving real-world scenarios such as interest, tax shopping sales, and so on.	
MP1. Make sense of problems and persevere in solving them.	Students solve multi-step and real-world percent problems.
MP3. Construct viable arguments and critique the reasoning of others.	Students recognize proportional relationships from non-proportional ones and discuss their reasoning with others.
MP4. Model with mathematics.	Students learn to represent proportional relationships as tables, graphs, verbal descriptions, diagrams and equations.
MP6. Attend to precision.	Students use units in their ratios requiring them to attend to the units such as 8 miles in 4 hours in a rate of 2 miles per hour.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
● If I multiply the decimal equivalent of a percentage by the whole, I can find the percent of a number. ● If set up a proportion, I can find the percent by putting the part/whole = x/100.	● Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*	
● I can use proportions to solve interest, sales tax, tipping, and other percentages problems.	

Common Student Misconceptions for this Unit

Students often have the misconception that percents cannot exceed 100% or 100 : 100. It is important that they become comfortable with percents such as 200% and can simplify the ratio from 200 : 100 to 2 : 1. Likewise 150% implies a ratio of 1.5 : 1 and 325% implies a ratio of 3.25 : 1.

Another common misconception is that students believe that if an original value decreases by a certain percentage every year, the original value will eventually become 0 when the percentages add to 100. For example, if they are told that the value of a car $10,000 and it decreases by 25% per year, then it will be valueless in 4 years because 25% x 4 = 100%. Students should be challenged to carry out the operation in which they realize the 25% always acts on the value of the car at any given year so that:

After year 1: $10,000 * .25 = $2500
After year 2: $2500 * .25 = $625
After year 3: $625 * .25 = $156.25
After year 4: $156.25 * .25 = $39.06

Indeed, students should be posed the question: “If this continues, will the car ever be valueless?”

Unit Assessment/Performance Task	DOK
Unit 3 Test Unit 3 Performance Task	

Vocabulary
● commission ● constant rate of proportionality ● equivalent ● fraction ● gratuities or commissions ● mark downs ● mark ups ● origin ● percent error ● percent increase/decrease ● proportion ● ratio ● simple interest

Key Learning Activities/Possible Lesson Focuses (order may vary)

Lesson Ideas

Using Proportions to Solve Real Life Percentage Situations

Students at this point should be able to use the 6th grade proportional curriculum understanding and this unit's lessons thus far to set up a proper proportion with percentages. Students will need to find the percent given a situation. In particular, real world situations of simple interest, taxes, sales pricing, tipping, and percent increase, or error are of importance.

a) Students compare measurements taken to true measurements in order to work with percent error. First students find the difference between the actual and recorded data and then find the percentage error.

b) Given real world situations students can determine the amount of interest, taxes, and discount or mark up in a given situation using proportions. Students should also be able to find the percentage given the original amount and the interest, tax, or discount or mark up. (For example finding the amount to leave on a restaurant bill for an 18% gratuity, taking a tip amount and determining the percentage of tip left on a given total, or taking the percentage gratuity and tip amount and determining the original bill).

Potential Lessons:
http://www.mathgoodies.com/lessons/percent/sale_price.html
http://www.mathgoodies.com/lessons/percent/interest.html
http://www.mathgoodies.com/lessons/percent/commission.html
http://www.mathgoodies.com/lessons/percent/sales_tax.html
http://www.mathgoodies.com/lessons/percent/change.html

Supplemental Materials and Resources

Online Lessons & Tasks

- Increasing and Decreasing Quantities by a Percent
  http://map.mathshell.org/materials/lessons.php?taskid=210&subpage=concept
- Ice Cream
  http://map.mathshell.org/materials/tasks.php?taskid=389&subpage=expert
- 25% Sale
  http://map.mathshell.org/materials/tasks.php?taskid=358&subpage=apprentice
- Buses
  http://map.mathshell.org/materials/tasks.php?taskid=365&subpage=apprentice
- Short Tasks - Ratios and Proportional Relationships
  http://map.mathshell.org/materials/tasks.php?taskid=397&subpage=novice
- Highway Robbery
  http://illuminations.nctm.org/LessonDetail.aspx?id=L838
- Now and Then
  http://illuminations.nctm.org/LessonDetail.aspx?id=L837
- What's Your Rate?
  http://illuminations.nctm.org/LessonDetail.aspx?ID=L511
- Ratio and Percents
  http://www.lausd.net/math/InstructionalGuides/Lessons/RATIO_AND_PERCENTS.pdf
- Problems with Percents
  http://www.lausd.net/math/InstructionalGuides/Lessons/PROBLEMS_WITH_PERCENTS.pdf
- Gauging Gas Mileage
  http://www.lausd.net/math/InstructionalGuides/Lessons/GAUGING_GAS_MILEAGE.pdf
- Calling Plans
  http://www.lausd.net/math/InstructionalGuides/Subjects/G7/2009%20MIG/CallingPlan%20Grade_7_LessonPlan.pdf

Worksheets

- Proportion Worksheet
  http://www.mathworksheets4kids.com/ratio/proportional.pdf
- Are the Ratios Proportional?
  http://www.teach-nology.com/worksheets/math/ratios/ratioandprop19.pdf
- Shopping with Proportions
  http://www.teach-nology.com/worksheets/math/ratios/ratioandprop24.pdf
- Unit Rates – Comparisons
  http://chatt.hdsb.ca/online%20conferences/mfm1p/Task1/pdfs/worksheet1-3.pdf
- Ratios and Unit Rates Worksheet
  http://www.shmoop.com/pre-algebra/handouts/ratios-percentages/ratios-and-unit-rates-questions.pdf
- Scale Drawings

http://www.ohs.osceola.k12.fl.us/staff/websites/pettettl/documents/GeometrySCALEDRAWINGSpp.174-180.pdf

Videos

- Unit Rates (Pearson)
  http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-890s.html
- Proportion and Direct Variation
  http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-196s.html

SMART Board Lessons

Online Interactive Activities & Games

- Ratios & Proportions Worksheets (online interactive)
  http://www.mathslice.com/actionctl.php?actionid=542
- Unit Rate Worksheet (online interactive)
  http://worksheets.tutorvista.com/unit-rate-worksheet.html
- Scale Factor
  - http://illuminations.nctm.org/ActivityDetail.aspx?ID=176

Literature connection:

"Many books and stories discuss comparative sizes; concepts of scale as in maps; giants and miniature people who are proportional to regular people; comparative rates, especially rate, especially rates of speed; and so on. A book may not appear to explore proportions, and the author many not have had that in mind at all, but comparisons are the stuff of many excellent stories and are at the heart of proportional ideas". (Van De Walle, J. (2004). *Elementary and Middle School Mathematics, Teaching Developmentally*. Fifth Edition. Boston: Pearson Education, Inc.)

Below are some suggestions:

If You Hopped Like a Frog, by David Schwartz, 1999

Counting on Frank, by Rod Clement, 1991

The Borrowers, Mary Norton, 1953

Interdisciplinary connections:

Science

- Scientists use ratio tables to find the amount of equivalent amounts are needed for greater or lesson quantities. Example if two Hydrogen atoms are present in 1 molecule of water then there are 14 water atoms in 7 molecules of water.
- The golden ratio and its appearance in nature. You can find the golden ratio in nature in some flowers, fruits, and vegetables.
- Allometry is the study of how these processes scale with body size and with each other, and the impact this has on ecology and evolution.

Social Studies/Geography

- Students can find percentages of demographic using data and vice versa.
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Tools/Manipulatives
● Graph Paper ● Rulers ● Calculator ● Common objects such as tennis shoes, cereal boxes, etc. ● Copies of restaurant menus
Suggested Formative Assessment Practices/Processes
● Teacher created quizzes ● Learning Log – where students access their understanding 1-5 (5 highest, 1 lowest) and then give a sentence or two about what did or did not understand. Use of three color cups (Green, Yellow, Red) for students to describe their level of understanding: Green cup = no problem, I understand what to do; Yellow cup= I need some assistance but it is not stopping me; Red cup= I need help

Differentiation and Accommodations
● Provide graphic organizers ● Provide additional examples and opportunities for repetition ● Provide tutoring opportunities ● Provide retesting opportunities after remediation (up to teacher and district discretion) ● Teach for mastery not test ● Teaching concepts in different modalities ● Adjust homework assignments