Grade/Subject	Grade 7 Accelerated Mathematics (grade 8 standards)
Unit Title	Unit 4: Irrational Numbers and Pythagorean Theorem (grade 8 standards)
Overview of Unit	This unit will provide a deeper understanding of the Pythagorean Theorem and its converse for students. They will apply the theorem to problems involving right triangles that model real world problems. They will also find distances between two points.
Pacing	Grade 7 Accelerated Mathematics 10 - 12 days (grade 8 standards)

Background Information For The Teacher
Students have solved equations in earlier grades and were introduced to rational and irrational numbers in grade 7. They did further application with rational and irrational numbers, including square roots of non-perfect squares, in Unit 1 of 8^{th} grade. This unit provides the real-life application of rational and irrational numbers by introducing students to the Pythagorean Theorem. This unit focuses on using the Pythagorean Theorem and its converse to represent the relationship between the lengths of the sides of a right triangle. Students have worked with right triangles in grade 5, where they identified the parts and properties of right triangles. In grade 6, students explored area and perimeter of right triangles. Prior to the CCSS, students were introduced to the Pythagorean Theorem in grade 7. With the Common Core, this unit will be the first time students are introduced to the Pythagorean Theorem. **Possible Teacher Misconceptions** The slogan “a squared plus b squared equals c squared” is an incomplete statement of the Pythagorean Theorem because there is no reference to a right triangle nor identification of the meaning of the variables. Here are preferred statements: ● For a right triangle, the sum of the squares on the legs is equal to the square on the hypotenuse. (A geometric focus) ● For a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. (A numerical focus) ● For a right triangle with legs of length a and b and hypotenuse of length c, a squared plus b squared equals c squared. (More precise restatement of the slogan)

<table>
<tr><th colspan="2">Essential Questions (and Corresponding Big Ideas)</th></tr>
<tr><td colspan="2">How can our understanding of the Pythagorean Theorem affect our understanding of the world around us?
● There are many practical applications of the Pythagorean Theorem, such as deconstructing larger shapes into right triangles to find the area of an irregularly shaped room.
Why is it necessary to prove formulas true?
● Students can use them and trust that they will get an accurate answer every time</td></tr>
<tr><th>Core Content Standards</th><th>Explanations and Examples</th></tr>
<tr><td>8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

Students learn that squaring and cubing numbers are the inverse operations to finding square and cube roots. This standard works with perfect squares and perfect cubes, and students will begin to recognize these numbers. Equations should include rational numbers such as $x^2 = \frac{1}{4}$ and $x^3 = 1/64$ and fractions were both the numerator and denominator are perfect squares or cubes.
$x^2 = \frac{1}{4}$

$\sqrt{x^2} = \pm\frac{\sqrt{1}}{\sqrt{4}}$

$x \pm \frac{1}{2}$

Square roots can be positive or negative because 2 X 2 = 4 and -2 x -2 =4.

What the teacher does:
● Introduce squaring a number and taking the square root as inverse operations, providing students opportunities to practice squaring and taking roots.
● Repeat the previous instruction for cubes and cube roots, also including</td><td>8.EE.2

Examples:

● $3^2 = 9$ and $\sqrt{9} = \pm 3$
● $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$
● Solve $x^2 = 9$
Solution: $x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

● Solve $x^3 = 8$
Solution: $x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

What the students do:
● Recognize perfect squares and perfect cubes.
● Solve equations containing cube and square roots.
● Discover and explain the relationship between square and cube roots and the sides of a square and the edges of a cube, respectively, by using hands-on materials.
● Reason that non-perfect squares and non-perfect cubes are irrational, including the square root of 2.</td></tr>
</table>

fractions where the numerator and denominator are both perfect cubes.

- Relate perfect numbers and perfect cubes of geometric square and cubes using square tiles and square cubes to build the numbers. A square root is the length of the side of a square, and a cube root is the length of the side of a cube.
- Encourage students to find patterns within the list of square numbers and then with cube numbers.
- Facilitate a class discussion around the question, "In the equation $x^2 = p$, when can p be a negative number?" Students should come to the conclusion that it is not possible.
- Discuss non-perfect squares and non-perfect cubes as irrational numbers such as $\sqrt{2}$.

Misconceptions and Common Errors:

It is important for students to have multiple opportunities and exposures with perfect cubes. This is a new concept in the curriculum and many students struggle with finding cube roots. A common misconception for cube roots is that any number times 3 is a perfect cube. Building larger cubes from smaller ones gives students a visual that they can rely on.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

There are many proofs of the Pythagorean Theorem. Students will work through one to understand the meaning of $a^2 + b^2 = c^2$ and its converse. The converse statement is as follows: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

What the **teacher** does:

- Explore right triangles to establish the vocabulary of hypotenuse and legs.
- Prepare graph chart paper so that each pair of students has one piece of chart paper with a right triangle drawn in the center. Triangles on different charts do not need to be the same size. Be sure that the side lengths of the triangle are whole numbers. Have students follow these steps:
  1. Find the lengths of the hypotenuse and the legs and mark them beside the triangle.
  2. Draw a square off of each side of the triangle.
  3. Determine the area of each square.
  4. Ask groups to write what they notice about the relationship of the areas of the squares.
- Facilitate a whole-class discussion to arrive at the conclusion that $a^2 + b^2 = c^2$. Facilitate a whole-class discussion about the converse of the theorem and pose the following question: Do you think the converse it true? Defend your answer.
- Enrich discussion with stories about the

8.G.6 Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.

What the **students** do:

- Use the correct vocabulary when writing or talking about the Pythagorean Theorem.
- Model a proof of the Pythagorean Theorem.
- Reason the converse of the Pythagorean Theorem is true as part of a class discussion.

Misconceptions and Common Errors:

Allow students to color code the triangle if necessary to see which square relates to which side of the right triangle they were provided. When stating the Theorem, many students miss the fact that the a^2 is the area of the square off a side *a,* not side *a* itself. Stress this fact clearly during the proof when students can see the square.

history of Pythagoras and he roots of the theorem

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Students solve problems where they must apply the Pythagorean Theorem. Problems may be real world or mathematical, and they may involve two or thee-dimensional situations.

What the **teacher** does:

- Model use of the Pythagorean Theorem to solve problems finding the unknown length of a side of a right triangle in two dimensions. "walk through" problems with students. Use examples where the solutions are not all whole numbers. Encourage students to draw diagrams where they can see the right triangles being used. Students may need a review of solving equations with square roots.
- Model use of the Pythagorean Theorem with problems in three dimensions. Use real world objects to help students see the right triangle in three-dimensions such as using a box where the right triangles can be drawn in space using string. Using technology to illustrate the three dimensionalities in some problems.
- Assign students problems to solve individually in pairs and in groups. Allow students to use any hands-on materials or technology they need to solve the problems. Justify solutions to the class.

8.G.7 Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.

What the **students** do:

- Solve problems using many different modeling techniques such as hands-on materials or technology. Problems may involve two or three-dimensional models.
- Use models with hands-on materials or technology to solve problems.
- Communicate with classmates using appropriate vocabulary.
- Check reasonableness of results to problems.

Misconceptions and Common Errors:

Common errors can be the result of students having difficulty with the computation. A review of computation with square roots may be needed.
Students with spatial visualization issues will have difficulty with the three-dimensional problems. Teachers need to model those problems with objects for these students more so than with the rest of the class. Allowing technology to model for these problems is also helpful.

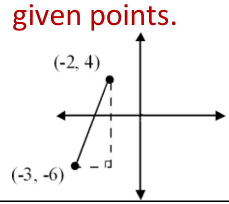
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Use the Pythagorean Theorem to find the distance between two points. Problems can best be modeled in a coordinate system.

What the **teacher** does:

- Present a real world problem to students where it is necessary to find the distance between two points. Allow students to struggle with it in pairs.
- Facilitate a class discussion about what the students tried and lead them to discover that using a coordinate plane

8.G.8 Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.



and the Pythagorean Theorem can be a solution strategy.

- Provide opportunities for students to solve problems where they must use the Pythagorean Theorem to find the distance between two points working individually, in pairs and in small groups. Students can present their work for critique by other students.

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Students expand their knowledge of the Real Number System to include irrational numbers. An irrational number is a decimal whose expansion does not terminate or repeat. Irrational numbers cannot be written in fraction form. Using decimal expressions, students compare rational numbers and irrational numbers to show that rational number expansions repeat and irrational number expansions do not. The notation "..." means "continues indefinitely without repeating. "For example,
$0.\underline{3}$ is a rational number that repeats but $\pi =$ $3.1415 \ldots$ does not repeat.

1. Let x = 0.555
2. Multiply both sides so that the repeating digits will be in front of the decimal. In this case, one digit repeats so both sides are multiplied by 10, giving 10x = 5.555...
3. Subtract the original equation form the new equation.
   10x = 5.555...
   <u>-x = 0.555...</u>
   9x =5
4. Solve the equation by dividing both sided of the equation by 9.
5. X = 5/9

<u>What the **teacher** does:</u>

<u>What the **teacher** does:</u>

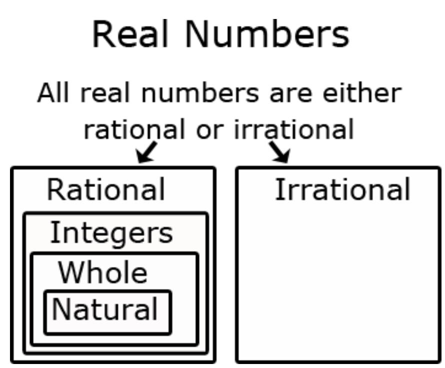
- Adopt the use of the Pythagorean Theorem to find the distance between two points on the coordinate plane.
- Solve real world problems using the Theorem as a strategy.
- Explain solution strategies using correct mathematical terminology and vocabulary.

<u>Misconceptions and Common Errors:</u>

This standard requires the use of several steps as a solution strategy. Some students have trouble when more than one step is involved. Walk these students through the steps, asking them questions along the way so they can make sense of the solution and not jut try to memorize the steps.

Some students use the Pythagorean Theorem to find missing sides for triangle that are not right triangles. To convince them that this will not work, have them reconstruct the proof with drawing squares on the sides of the triangle that is not right. They will see that the areas do not add up.

8.NS.1 Students can use graphic organizers to show the relationship between the subsets of the real number system.



- Pose questions such as the following: "Will a rational number eventually repeat?" Can you find a rational number that does not repeat? Use this discussion as an introduction for students to discover irrational numbers.
- Have students reason about the inclusive nature of the subsets of Real Number System and complete a Venn diagram of the Real Number System.
- Access prior knowledge about converting fractions to decimals. Relate the concept to changing the decimal expansion of a repeating decimal expansion of a repeating decimal into a fraction and a fraction into a repeating decimal. Provide examples.

<u>What the **students** do:</u>

- Clarify understanding of rational numbers as repeating or terminating through discussion about irrational numbers.
- Recognize and use the notation for decimal expansions of irrational numbers.
- Complete a Venn diagram to clarify their understanding of the Real Number System as the set of numbers made up of the rational and the irrational numbers.
- Convert decimal expansions into equivalent fractions using an algorithm.
- Use strategies other than conversions for some decimal expansions; for example, after exploring the ninths, students may remember the repeating pattern
- $\left(\frac{1}{9} = 0.\underline{1},\ \frac{2}{9} = 0.\underline{2},\right)$
- Recall common fractions such as ¾ = 0.75.

<u>Misconceptions and Common Errors:</u>

Some students have difficulty understanding the relationship of the subsets of the Real Number System with a Venn diagram. Try hands–on approach using boxes or bags that fit inside one another to represent the subsets.

Some students need more practice than others converting repeating decimals to equivalent fractions. This can be done over time with mini-practice sessions weekly.

8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., □^{2}). *For example, by truncating the decimal expansion of √2, show that √2 is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Students compare irrational numbers and locate them on a number line by finding their rational approximations. Find rational approximations by creating lists of numbers by answering the following question: Between which two numbers will you find $\sqrt{2}$? Since 1^2 = 4, it is between 1 and 2. To be more precise, is it closer to 1 or 2? Systematically square 1.1, 1.2, 1.3, 1.4... 1.9. Between which two numbers

8.NS.2 Students can approximate square roots by iterative processes.

Examples:

- Approximate the value of $\sqrt{5}$ to the nearest hundredth.

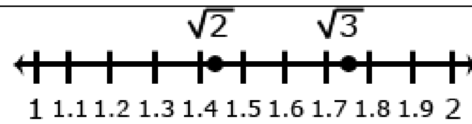
Solution: Students start with a rough estimate based upon perfect squares. $\sqrt{5}$ falls between 2 and 3 because 5 falls between 2^2 = 4 and 3^2 = 9. The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. $\sqrt{5}$ falls between 2.2 and 2.3 because 5 falls between 2.2^2 = 4.84 and 2.3^2 = 5.29. The value is closer to 2.2. Further iteration shows that the value of $\sqrt{5}$ is between 2.23 and 2.24 since 2.23^2 is 4.9729 and 2.24^2 is 5.0176.

- Compare √2 and √3 by estimating their values, plotting them on a number line, and making comparative statements.

do you find 2? Repeat the process until you have the degree of precision you are seeking.

<u>What the **Teacher** does:</u>

- Pose the following question: Where should you place 3.14159... on the number line? Provide a number line to fill in with rational numbers. Provide several integers and fractions along with 3.14159... and allow them to reason where they will place 3.14259... Discuss how precise they were in their placement. Did they place is between 3 and 4 or between 3.141 and 3.142? Is between 3 and 4 precise enough? Through this questioning, lead students to conclude that approximations of irrational numbers are used to compare and locate them on a number line.
- Allow students to compare the size of irrational numbers based on their location on the number line. Highlight the reasoning students already use to compare the size of integers, fractions, and so on based on distance and direction on a number line.
- Provide opportunities for students to practice approximating irrational numbers. Be sure to provide instructions on how precise the approximation should be, such as, "To 3 decimal places."
- Provide students opportunities to explaining in writing how to get more precise approximations for irrational numbers. For example, they could write a letter to next year's seventh-grade class explaining how to approximate the
- $\sqrt{5}$ to three decimal places.



Solution: Statements for the comparison could include:

√2 is approximately 0.3 less than √3

√2 is between the whole numbers 1 and 2

√3 is between 1.7 and 1.8

<u>What the **students** do:</u>

- Reason abstractly to determine where to place an irrational number on the number line. Students begin to focus on the precision required of the task. It is not unreasonable to expect them to ask how precise they should be for the given exercise. Tenths? Hundredths?
- Look for and express regularity in the repeated reasoning used in finding approximations of irrational numbers.
- Reason abstractly as they become more familiar with the process to find approximations of irrational numbers to streamline the algorithm.
- Express thinking in writing to clarify understanding about how to find precise approximations of irrational numbers.

<u>Misconceptions and Common Errors:</u>
When rational numbers written in decimal form have more than three digits that repeat, some students stop the division process and call it an irrational number. These students need to be encouraged to preserve with the division until they are convinced there is no repeat. These students may not have a clear understanding of rational numbers as numbers that can be written in fraction form. This fact should be made explicit during instruction. To help students who become overwhelmed with the process to approximate irrational numbers, suggest an organized format. For example, set up three columns with questions that need to be answered for each. Some student my need the template at first.

_____falls between which two whole numbers?

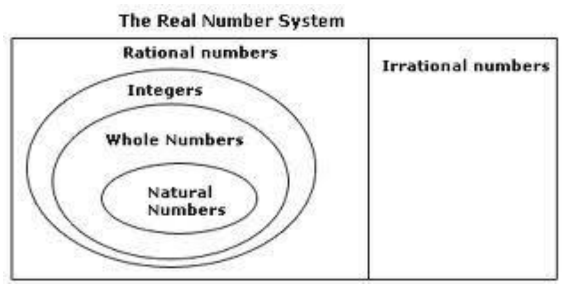
Is_____closer to_____or_____?

Is_____closer to_____or_____?

Standards for Mathematical Practice	**Explanations and Examples**
Work with radicals and integer exponents. **8.EE.2,** In this cluster students learn how to compute with integer exponents. Students build on what they learned about what they learned about square roots to solve equations.	
MP5. Use appropriate tools strategically.	
MP6. Attend to precision.	
Understand and apply Pythagorean Theorem **8.G.6, 8.G.7, 8.G.8** This cluster focuses on introducing the Pythagorean Theorem. Students explore the relationship between the sides of a right triangle to understand the formula $a^2 + b^2 = c^2$ and then solve problems applying the theorem. **MP3. Construct viable arguments and critique the reasoning of others.**	
MP4. Model with mathematics.	Students model an informal proof to understand the Pythagorean Theorem.
MP6. Attend to precision.	Students use modeling to understand the meaning of the Pythagorean Theorem.
MP7. Look for and make use of structure.	Students check their results to all computations. Students look for patterns in right triangles to help solve problems.

<table>
<tr><th colspan="2">K-U-D</th></tr>
<tr><th>KNOW
Facts, formulas, information, vocabulary</th><th>DO
Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)</th></tr>
<tr><td>

- There are numbers that are not rational called “irrational”.
- Irrational numbers are a subset of the Real Number System.



Every number has a decimal representation:

- Irrational decimals are non-repeating and non-terminating
- Rational number decimals eventually terminate or repeat.
- Irrational numbers can be approximated for comparing and ordering them.
- A perfect square is a number in which the square root is an integer.
- A perfect cube is a number in which the cube root is an integer.
- $\sqrt{2}$ is irrational.
- Equivalent forms of an expression allows for efficient problem solving.
- Estimation can be used as a means for predicting & assessing the reasonableness of a solution.

</td><td>

- KNOW rational and irrational numbers
- UNDERSTAND decimal expansion
- SHOW decimal expansion repeats
- CONVERT repeating decimal expansion to a rational number
- USE
  - rational approximations of irrational numbers
    - COMPARE sizes of rational numbers
    - LOCATE rational numbers approximately on a number line
    - ESTIMATE value of expressions
  - square root and cube root symbols
- EVALUATE
  - square roots of perfect squares
  - cube roots of perfect cubes
- APPLY the Pythagorean Theorem.
- DETERMINE unknown side lengths in right triangles.
- FIND distance between two points in a coordinate system.
- EXPLAIN a proof of the Pythagorean Theorem and its converse.
- USE square root and cube root symbols.
- REPRESENT solutions to equations using radical symbols.
- EVALUATE square roots of small perfect squares and cube roots of small perfect cubes.

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● Vocabulary: see section below ● In a right triangle, the "legs" are the side lengths that form the right angle and the "hypotenuse" is the diagonal length that connects the legs. ● Pythagorean Theorem: $a^2 + b^2 = c^2$, where a and b are the legs of the triangle and c is the hypotenuse. ● Proof of the Pythagorean Theorem ● Converse of the Pythagorean Theorem ● The Pythagorean Theorem only applies to right triangles. ● Pythagorean triples, such as (3, 4, 5) and (5, 12, 13). ● $\sqrt{2}$ is irrational	

UNDERSTAND

Big ideas, generalizations, principles, concepts, ideas that transfer across situations

In the real number system, numbers can be defined by their decimal representations.

- Students will understand that:
    - The Pythagorean Theorem is essential for solving real world problems because it can be applied to many situations where a missing length of a right triangle needs to be calculated.
    - The Pythagorean Theorem relates the areas of squares on the sides of the right triangle.
    - The Pythagorean Theorem is a relationship among the sides of a right triangle.
    - There is more than one way to prove the Pythagorean Theorem and its converse.

Common Student Misconceptions for this Unit

- Students may not continue to divide to recognize a repeating decimal.
- Students may think that squaring means to multiply by 2.
- Students may think that finding the square root means to divide by 2.
- Students may think that cubing means to multiply by 3.
- Students may think that finding the cube root means to divide by 3.
- Students do not consider the negative square roots of the number. For example, $\sqrt{9} = +3$ instead of $\sqrt{9} = \pm 3$
- On a number line, students may struggle with plotting negative values. For example, -1.5 would be placed between $0\ and - 1$.
- On a number line, students will struggle with comparing negative numbers to negative numbers. For example $-6 > -5$

- Students may divide the denominator by the numerator (instead of numerator by denominator) when converting a fraction to a decimal), resulting in a number greater than one.
- Students might believe that…
    - $ab + ab = a^2 + b^2$
    - $(a + b)^2 = a^2 + b^2$
    - a, b, and c are interchangeable when referring to the sides of a right triangle, instead of identifying the hypotenuse as c.
    - The Pythagorean Theorem applies to all triangles.

Unit Assessment/Performance Task	**DOK**
Unit 4 Test: Irrational Numbers and Pythagorean Theorem Patterns in Prague Performance Task	

Vocabulary
- Cartesian Coordinate Plane   - Converse   - Hypotenuse   - Irrational numbers   - Leg   - Perfect Square   - Pythagorean Theorem   - Radical   - Real numbers   - Rational numbers   - Natural numbers   - Whole numbers   - Whole numbers   - Integers   - Irrational numbers   - Repeating decimals   - Terminating decimals   - Decimal expansion   - Square root   - Perfect square   - Cube root   - Perfect cube   - Approximate   - Rational numbers   - Right triangle   - Similar figures   - Square root
Key Learning Activities/Possible Lesson Focuses (order may vary)
These are ideas for lessons. **Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)** Use Station 1 activity to group numbers into terminating, repeating, or "other" decimals, discuss different types of numbers, especially square roots; HW: find 3 square roots that are terminating and 3 that are not *In cooperative groups, students will develop the definition of rational and irrational numbers by investigating sets of numbers. (integers, naturals, whole, rational, irrational)*

- Understand and show the relationship between the subsets of the real number system (rational and the subsets of rational numbers and irrational numbers and irrational numbers)
- Identify and know which numbers belong to which subsets of the real number system
- Show decimal expansion of a rational number repeats or terminate
- Convert decimal expansions (both repeating and terminating) to a rational number

Introduce real number system using graphic organizer, then have students take a given number and put it on giant number system; "exit ticket" for homework

Teachers will provide students with the area of various size squares and students will find side lengths.

- Understand and know perfect squares and square roots (up to 144, and including 225, 400, and 625) and that they are part of the real number system
- Estimate the rational value of square roots (using mental math)
- Using square root symbols, represent and evaluate solutions to equations

Students will use knowledge of square roots to devise a definition of cube roots. With this definition, students will identify 2 cube roots.

- Understand and know perfect cubes and cube roots (including 1 ,8 , 27, 64, 125, 1000) and that they are part of the real number system
- Using cube roots symbols, represent and evaluate solutions to equations

Compare and locate various types of Rational and Irrational numbers on the number line (exact and estimated)
Using real world examples (baseball statistics, Olympic results, etc.) students will compare, order, and locate on the number line.

- Compare sizes of rational numbers
- Use rational approximations of irrational numbers such as the
- $\sqrt{2}$ and $\sqrt{3}$, locate on the number line, and make comparative statements
- Locate positive and negative rational and irrational numbers exactly and approximately on the number line.

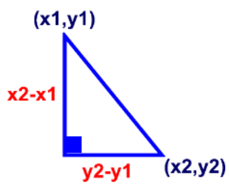
These are ideas for lessons.
Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed) Introduction to and Discovery of the Pythagorean Theorem.

- Provide various methods for students to discover the Pythagorean Theorem.
- Have students describe vocabulary using their own words and a picture (leg, hypotenuse, converse, Pythagorean triples, Pythagorean theorem) – Possible methods for teaching vocabulary: The Frayer model, two column notes, Marzano's 6 steps to Vocabulary Mastery
- Have students complete the Proofs of the Pythagorean Theorem handout in pairs or small groups. http://map.mathshell.org/materials/download.php?fileid=804

  Application of the Pythagorean Theorem
- Have students use the theorem to find the missing lengths of various triangles independently or in pairs. (For struggling students, have students only find the hypotenuse first. Then, have students find a missing leg.) Provide students with various examples, some of which should be drawn on a coordinate plane.
- Initiate the discussion with students about the importance of precision when dealing with a, b, and c. Emphasize to students that a and b ALWAYS represent the legs of the triangle and c ALWAYS represent the hypotenuse. Discuss how a and b can be interchangeable with each other.
- Have students apply their knowledge of the Pythagorean Theorem to real life applications. Provide students with word problems where they are required to sketch a diagram and solve for the unknown side length. Make sure students are able to distinguish between the legs and the hypotenuse.
- Have students find the missing side of each triangle from the above examples. Make sure to provide multiple opportunities for students to find missing hypotenuses and legs on triangles with various orientations.
- Have students complete formative assessments throughout the lessons. Some examples are below.
  - i. Frank Rd and James Rd. make a perpendicular intersection. The state wants to build a new road. The new road will intersect 3 miles north of the intersection on Frank Rd. and 4 miles west of the intersection on James Rd. How long will the new road be that intersects Frank and James Rd?
  - ii. The mobile phone company is anchoring wires to the top of their 1200 ft high communication towers. The cable for the support wire comes in a roll that is 1950 ft long. The company requires you to use the entire roll. How far from the base of the tower do they need to be anchored?
  - iii. Jane's TV http://map.mathshell.org/materials/download.php?fileid=1098
  - iv. Revisit proofs of the Pythagorean Theorem

Real life application of the **converse** of the theorem (How do you know if a given triangle is a right triangle?)

- Provide students with various triangles and have them apply the Pythagorean Theorem to determine if the triangles are or are not right triangles. (Ex. Written problems, manipulatives, word problems, pictures on a coordinate plane, etc.) Consider having this done in small group.
- Establish the connection to the Distance Formula for students: The Pythagorean Theorem serves as the basis for determining the distance between two points in the Cartesian plane, called the Distance Formula. Using the following example, have students derive the distance formula by providing students with the diagram on a coordinate plane. Triangles should be drawn in any of the four quadrants to demonstrate to students that the formula is valid regardless of the value of x and y.



$$\text{distance between points} = \text{hypotenuse}$$
$$\text{distance}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- If (x_1, y_1) and (x_2, y_2) are points in the plane, then the distance between them can be determined using the Pythagorean Theorem.

Section 7.1

Day 1: Inquiry lab from Glencoe book to introduce theorem (HW: finish activity)

Day 2: Teach Pythagorean Theorem and converse (HW: p. 16-17)

Day 3: Activity - "How Accurate Is It" (HW: p. 16-17)

Day 4: Put it together - real-world problems

Day 5: Quiz

Section 7.2

Day 1: Introduce distance formula

Day 2: Complete the "town" activity

Supplemental Materials and Resources

Literature connection:

- *The Square Root of 2* By: David Flannery
- *Square Root* By: Derek Beaulieu
- Short task on the number system: http://map.mathshell.org/materials/tasks.php?taskid=398#task398
- Lesson on Repeating Decimals: http://map.mathshell.org/materials/lessons.php?taskid=421#task421
- Pop up game- Rational or Irrational Number?: Http://www.quia.com/pop/37541.html?AP_rand=1107411821
- Decimal Expansion: http://mathworld.wolfram.com/DecimalExpansion.html
- Properties of Real Numbers: Http://www.math.com/school/subject2/lessons/S2U2L1Dp.html
- When not knowing math can cost you $15,000: http://www.youtube.com/watch?v=BbX44YSsQ2I
  This is a clip from a television game show where the contestant answers a question about square numbers incorrectly and loses $15000.

Tools/Manipulatives

- Graph paper
- Calculator
- Computer access (if possible)
- Ruler
- Geometer's Sketchpad/Geogebra
- Pattern blocks/trangrams
- Sticky notes
- number line

Suggested Formative Assessment Practices/Processes

Teacher created quizzes

Exit slips suggestions that can be used throughout the unit:

a. Solving for a missing side of a triangle
b. Solve for missing side when the triangle is a Pythagorean triple
c. Given certain side lengths determine if the triangle is a right triangle
d. Determine square roots of perfect squares
e. What is the relationship between the three sides of a right triangle?
f. Why is it important to prove the Pythagorean Theorem in more than one way?

g. Use the Distance Formula to find the length of a line between two points on the coordinate plane.

Journal writing:

a. Begin or end class with a few minute writing assignment. These should provide a one to one connection between the student and teacher. http://www.teachingenglish.org.uk/articles/learner-diaries
b. Provide students with a question to respond to or allow it to be an open ended option. For example "When might you use the Pythagorean Theorem in your everyday life?", "Identify a common mistake that you think students make when they use the Pythagorean Theorem?".

Gallery walks:

a. As students do diagrams and sketches in this unit, display student work throughout classroom for gallery walks, i.e. proofs at beginning of unit, problems where students verified the converse of the Theorem etc.
b. Have students use sticky notes and write comments or questions and post them on other student work.
c. Use sticky notes with comments and questions as the subsequent day's initiation.

Videos of student work

a. Have students video their work on the Pythagorean Theorem
    - Teaching the proof of the Pythagorean Theorem
    - Explaining the converse of the Theorem
    - Outlining the steps to find a missing side of a right triangle
    - Outlining the steps to use the Theorem in a real life problem

Smart Board Lesson

a. Have students create a smart board lesson
    - teaching the proof of the Pythagorean Theorem
    - Explaining the converse of the Theorem
    - Outlining the steps to find a missing side of a right triangle
    - Outlining the steps to use the Theorem in a real life problem

Differentiation and Accommodations
● Provide graphic organizers ● Provide additional examples and opportunities for repetition ● Provide tutoring opportunities ● Provide retesting opportunities after remediation (up to teacher and district discretion) ● Teach for mastery not test ● Teaching concepts in different modalities ● Adjust homework assignments