Grade/Subject	Grade 7 Accelerated Mathematics
Unit Title	Grade 7A Unit 6 Volume of Cylinders, Cones, and Spheres (grade 8 standard)
Overview of Unit	Within this unit, students will solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. Students will discover the formulas for finding the volume of cones and spheres. Students will also relationship to the volume of a cylinder.
Pacing	Grade 7A Mathematics: 8 - 10 days

Background Information For The Teacher
Students start using volume of right rectangular prisms in grade 5. In grade 6, they add in rational values for side lengths. Students build on their knowledge of area of circles and volume of prisms and pyramids in grade 7 (7.G.4 and 7.G.6). This unit will extend their understanding of volume to include cones, cylinders, and spheres and the relationships between those figures. Students will also need to use the Pythagorean Theorem which is covered in Unit 5 of grade 8.

Essential Questions (and Corresponding Big Ideas)

How do you find the volume of three-dimensional figures?
How are the volumes of cylinders, cones, and spheres related?

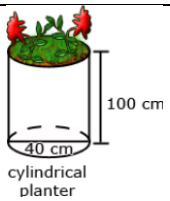
- Solids such as cylinders, cones, and spheres are all around us. You can find the volume of these figures to solve real-world problems.
- The volume of any three-dimensional figure is dependent on the area of its base, height and the shape and number of parallel bases (layers) that the figure has.
- The volume of a cone is 1/3 of the volume of a cylinder (with same radius and height.)
- The volume of a sphere is 4/3 the volume of a cylinder (with same radius and height).

Core Content Standards	**Explanations and Examples**
8.G.9 Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems This standard has two distinct parts. First, students learn the volume formula for cones, cylinders, and spheres. Then they apply this knowledge to solve real-world and mathematical	8.G.9 Example: James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.

problems. The formulas should be taught through experiments where students figure out the formulas.

What the Teacher does:

- Prepare students to discover the formulas through hands-on experiences. For the volume of cylinder, compare it to what students already know about the volume for a right rectangular prism. Using the knowledge that the volume of a right rectangular prism is the area of the base times the height, compare a cylinder to a prism of the same height. Have physical models on hand. Note that the base of the prism is a rectangle, and the base of the cylinder is a circle. Lead students to reason that the formula for the volume is the same as the volume of a right rectangular prism in that both are the area of the base times the height. So far the cylinder, it is $V = h\pi r^2$, which is the base area (πr^2), times the height area (h).
- Compare the volume of a cylinder to a cone to derive the formula for the volume of a cone. Use a cylinder and cone of equal heights. Let students fill the cone with rice or water and ask students to estimate how many times they need to fill the cone in order to fill the cylinder. Students will conclude that the cone holds 1/3 the volume of the cylinder of the same height, thus the formula $V = \frac{1}{3}h\pi r^2$.
- Compare the volume of a sphere to the volume of a cylinder of the same height. Model half a sphere placed inside the cylinder of same height and base. The area of the base of the cylinder and the area of the section created by cutting the sphere in half are both πr^2. In this model, the height of the cylinder is also r, so the volume of the cylinder is πr^3. Fill the hemisphere with rice or water and estimate how many will fill they cylinder. The volume of the hemisphere with radius r is 2/3 that of the cylinder. Since the hemisphere is only half of the sphere, we double and find $V = \frac{4}{3}\pi r^3$.
- Provide opportunities for students to explain in writing how they understand the volume formulas for cones, cylinders, and spheres.
- Prepare a variety of problems and settings for students to solve real-world and mathematical problems by applying the volume formulas for cones, cylinders, and spheres.



cylindrical planter

What the Students do:

- Participate in experiments that help them derive the volume formulas for cones, cylinders, and spheres.
- Explain in writing their understanding of the volume formulas for cones, cylinders, and spheres.
- Solve mathematical and real-world problems that involve finding the volumes of cones, cylinders, and spheres.

Misconceptions and Common Errors:

Students may confuse the three formulas if they are asked to memorize them without any understanding of why they make sense. It is important to spend time on the derivations and have students physically take part and make the discoveries for themselves.

Standards for Mathematical Practice	**Explanations and Examples**
Solve real-world and mathematical problems involving of cylinders, cones and spheres. **8.G.9** This cluster focuses on knowing and applying volume formulas for cylinders, cones, and spheres and then applying this knowledge to solving problems. **MP1. Make sense of problems and persevere in solving them.** **MP4. Model with mathematics.** **MP6. Attend to precision.** **MP7. Look for and make use of structure.**	Students solve problems involving volume of cones, cylinders, and spheres. Students use modeling to understand the meaning of the Pythagorean Theorem. Students check their results to all computations. Students look for patterns in right triangles to help solve problems.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
● Formulas for volume of cones, cylinders, and spheres. ● Volume is measured in cubic units. ● The volume of a cylinder is the area of the base multiplied by the height. ● The volume of a cone is 1/3 of the area of the base multiplied by the height. ● The volume of a sphere is 4/3 times π multiplied by the radius cubed.	● KNOW the formulas for volume of cones, cylinders, and spheres. ● USE the formulas for volume of cones, cylinders, and spheres to solve real-world problems. ● Find the volume of cones, cylinders, and spheres within the context of a problem.

UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*
● The volume of any three-dimensional shape is dependent on the area of its base, height of the shape and the number of parallel bases (layers) that the shape has. ● The relationship between the volumes of cylinders, cones, and spheres. ● The volume of a cone is 1/3 of the volume of a cylinder (with same radius and height). ● The volume of a sphere is 4/3 the volume of a cylinder (with same radius and height).
Common Student Misconceptions for this Unit
Students may struggle with unit conversions. Students may struggle labeling the information on a graphic given in a problem (ie. interchanging the diameter and the radius). Students will assume the tallest container always has the greatest volume (ie. greatest height equals greatest volume). Students may struggle with multiplying by 4/3 to compute the volume of a sphere.

Unit Assessment/Performance Task	**DOK**
Unit 7 Test Unit 7 Performance Task “Pablo’s Icy Treat Stand” Unit 7 Performance Task “Flower Vases”	

Vocabulary
Cone Cubed (raised to 3rd power) Cubic Units Cylinder Diameter Height Hemisphere Pi (π) Radius

Sphere
Squared (raised to 2nd power)
Volume

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Activity One – Finding Volume of a Cylinder
In this activity students must work together to determine the volume of multiple cylinders for use as a soda can and justify which one they would use if they were the manufacturer. Within this activity, students must find and compare the volumes of three different cylinders.

Activity Two – Discovering Volume of a Cone
In this activity students must work together to discover the relationship between the volume of a cone and a cylinder. Students will use a cylinder and a filler material to determine the formula for the volume of a cone.

Activity Three – Discovering Volume of a Sphere
In this lesson students must work together to discover the relationship between the volume of a sphere and a cylinder. Students will use a cylinder and a filler material to determine the formula for the volume of a sphere.

Supplemental Materials and Resources

Literature connections:
This activity draws on the novel Holes by Louis Sachar (New York: Farrar, Straus and Giroux, 1998). The National Council of Teachers of Mathematics. (2008) *Navigating through mathematical connections in grades 6–8.* Reston, VA. NCTM.

Interdisciplinary connections:
Science:
Determine appropriate use of beakers; measuring quantities of liquids; chemicals by volume.
http://science-notebook.com/measure02-vol.html

Economics:

Determining the price of food by volume; shipping products by volume; determine size of storage tank to minimize land use.
Tools/Manipulatives
Three-dimensional models of cylinders, cones, and spheres Rice, sand, water, beans, etc. (Some type of filler material to show volume) Protractor Compass Straightedge Calculator Stiff/construction paper
Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes

Differentiation and Accommodations
● Provide graphic organizers ● Provide additional examples and opportunities for repetition ● Provide tutoring opportunities ● Provide retesting opportunities after remediation (up to teacher and district discretion) ● Teach for mastery not test ● Teaching concepts in different modalities ● Adjust homework assignments