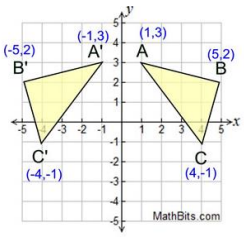
Grade/Subject	Grade 7 Accelerated Mathematics (grade 8 standards)
Unit Title	Unit 7: Congruence and Similarity
Overview of Unit	Understand congruence and similarity using physical models, transparencies, or geometry software
Pacing	Grade 7 Accelerated Mathematics: 21 - 25 days

Background Information For The Teacher
Rationale In 7th grade, students reasoned about relationships among two-dimensional figures using scale drawings and informal geometric constructions to gain familiarity with the relationships between angles formed by intersecting lines. In this unit, students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students will show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. **Key Learning** The primary focus of this unit is to verify that two figures are congruent or similar using transformations. Students should recognize that a reflection, rotation, and translation are rigid transformations that maintain congruence and a dilation is a transformation that enlarges or shrinks an object but maintains its shape. A secondary focus of this unit is to understand the relationships that exist between the angles created when parallel lines are cut by a transversal. (It has been decided that the secondary focus will be introduced in 7th grade along with other angle properties. However it is assessed on the 8th grade SBAC so it is important to review these concepts with students.) **Displaced Concepts** Prior to the CCSS, students did not encounter properties of angles created by two parallel lines cut by a transversal until Geometry. Also prior to the CCSS, students were introduced to transformations in grade 6 and grade 7. In the CCSS, students have graphed two-dimensional shapes on a coordinate plane in grade 6 and have worked with scale drawings and adjacent angles in grade 7. Students were also introduced to the concepts of two figures being congruent or similar in grade 7. Students will be introduced to transformations in grade 8 and will continue to build on their understanding in high school Geometry.

Changes from Past Practice
In the CMT strands, students were required to rotate a figure about a point of the figure. In the CCSS, students are now required to rotate a figure about any fixed point (center of rotation). In the CMT strands, students were not required to dilate a figure, as they are now required to do in the CCSS.

Essential Vocabulary for the Teacher (if any)
Rigid motion: A transformation of points in space consisting of a sequence of one or more translations, reflections and/or rotations. Rigid motions are here assumed to preserve distances and angles measures.

Essential Questions (and Corresponding Big Ideas)	
How can you show two figures are congruent or similar using transformations? ● A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. ● A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations.	
Core Content Standards	**Explanations and Examples**
8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. Eighth graders add rotations, reflections, and translations to their study of transformations from Grade 7 dilations. Students verify through experimentation with figures on a coordinate plane that lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure, and parallel lines are taken to parallel lines. This standard is an introduction, and students should spend time exploring these transformations. See picture below.	8.G.1 Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated. Students are not expected to work formally with properties of dilations until high school.



What the Teacher does:

- Provide basic exercises for the students to learn about rotations (turns), reflections (flips), and translations (slides) on the coordinate plane. Use hands-on materials such as shapes cut from paper to have students model the transformations before using the coordinate grid. Wallpaper patterns provide excellent models of rotations, translations, and reflections. Transparencies also illuminate the transformations. Students can go online to find examples of the transformations in art and architecture.
- Isolating each transformation, facilitate a class discussion about what they notice about the new figure compared to the original. They should notice that lines are taken to lines, angles to angles of the same measure, and parallel lines to parallel lines. Use the correct mathematical notation of A and A' (A' read as "A prime?") as the labels for the transformation and the original figure.
- Give students opportunities to identify the transformation(s) that occurred to take one figure to another. One activity is to let each student transform a figure on the coordinate plane using one, two or three transformations. Then have the students trade papers to see if they can identify the transformations used by their classmates.

8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a

What the Students do:

- Accurately transform figures on the coordinate plane using rotations, reflections, and translations, and the correct notation.
- Identify the transformations used to transform one figure into another using hands-on materials and on the coordinate plane.
- Discover that for a transformation, lines are taken to lines, line segments to line segments, angles to angles of the same measure, and parallel lines to parallel lines.

Misconceptions and Common Errors:

Students with spatial visualization problems will find this standard difficult. These students need much practice. The use of technology to show the transformations can help as the student can experience many transformations in a shorter period of time than if each had to be done by hand. There are numerous YouTube videos that deal with transformations.

sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Students use what they previously learned about transformations to determine congruency between figures. Congruent figures share the same size and shape. When given two congruent figures, students describe the sequence of transformations that occurred to create the congruent figure. Note that dilations cannot be used for congruent figures.

What the Teacher does:

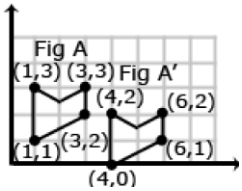
- Have students use one or two transformations to transform a figure. Display the student work to use as your example to of congruent figures. Use the display to lead students to discover the definition of congruent.
- Provide students with examples of congruent figures and ask them to trace the sequence of transformations that created the congruent figure. Check the students are using the correct notation for the figure and the original of A and A' (A' read as "A prime).
- Present students with opportunities to write about congruent figures; for instance, trade examples of congruent figures created by students and the original figures. Have students justify in writing why the new shape is congruent to the original using appropriate vocabulary.

8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

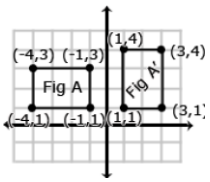
Students continue looking at two-dimensional figures on the coordinate plane, concentrating on the coordinates of the resulting figure after transformations, including dilations learned in Grade 7.

8.G.2. Examples:

• Is Figure A congruent to Figure A'? Explain how you know.



• Describe the sequence of transformations that results in the transformation of Figure A to Figure A'



What the Students do:

- Discover that a series of transformations can create a figure that is congruent to the first Communicate the understanding through writing.
- Create congruent figures by applying a series of transformations.
- Use correct notation in labeling congruent figures.

Misconceptions and Common Errors:

Students with spatial visualization issues may have difficulty with this concept. Color coding lines and angles in the congruent figure to match the original may help.

8.G.3.

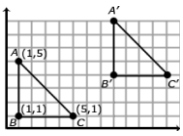
Dilation: A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale

What the Teacher does:

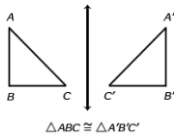
- Display several images (figures after transformation) and pre-images (original figures). Use different polygons. Focus student attention on the coordinates and pose questions so that students develop a sense of how the coordinates change during different transformations.
- Concentrate on moving one pre-image by adding, subtracting, and multiplying the coordinates such as add 2 to x, subtract 3 from y, combinations of changes to x and y.
- Then ask students questions such as the following: What happened to the figure? Did the change affect just one vertex or all? In what way?
- Facilitate students' generalizing about the changes that preserve size and/or shape. Note that dilations create distortions of the pre-image.
- Provide opportunities such as journals and exit slips for students to describe transformations with both words and coordinate points.
- Present examples to students for this standard not only on paper but using technology.

factor. In dilated figures, the dilated figure is *similar* to its pre-image.

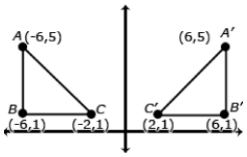
Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. Δ*ABC* has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B + C also move in the same direction (7 units to the right and 3 units up).



Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.



When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.



Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360˚. Rotated figures are congruent to their pre-image figures.

Consider when Δ*DEF* is rotated 180˚ clockwise about the origin. The coordinates of Δ*DEF* are D(2,5), E(2,1), and F(8,1). When rotated 180˚, Δ*D'E'F'* has new coordinates

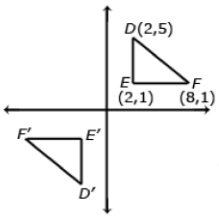
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

With this standard students move from congruence to similarity. Students develop the understanding that similar figures can be created by a series of transformations, including rotations, reflections, dilations, and translations, and can identify those transformations given an image and pre-image.

What the Teacher does:

- Display two images and pre-images. One should show congruent figures and the other should demonstrate similar figures. Use these to help students understand the difference between the two concepts.
- Present students with pre-images and images of similar polygons and trace, with students, the sequence of transformations that produced the image. Challenge students to find the most efficient combination of transformations. Give students practice doing the same using paper and pencil examples, but also those found in the real world such as examples from art, architecture, and the natural world.
- Provide opportunities for students to communicate their reasoning through vehicles such as journals and exit slips.

D'(-2,-5), E'(-2,-1) and F'(-8,-1). Each coordinate is the opposite of its pre-image.



What the Students do:

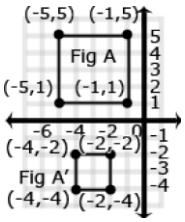
- Discover that transformations change the coordinates of images from the pre-images in specific ways.
- Analyze images and pre-images to determine the transformations that took place and explain using words and coordinate points.

Misconceptions and Common Errors:

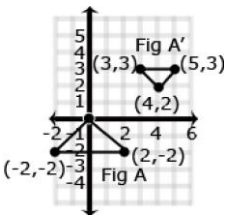
Allowing students to use rulers to follow the lines from pre-image to image will help students with tracking difficulties.

8.G.4. Examples:

• Is Figure A similar to Figure A'? Explain how you know.



• Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



What the Students do:

- Model similar figures from pre-imaging using transformations.
- Trace the sequence of transformations that create a given image from a give pre-image.
- Communicate orally and/or in writing the transformation sequence found for creating a particular image from a pre-image.

8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
(This standard will be included in grade 7 but should be reviewed in grade 8.)

Students are expected to make informal arguments while exploring facts about the sum of the angles of a triangle, exterior angles of triangles, angles created when parallel lines are cut by transversal, and the angle-angle criterion for similar triangles. Note that formal two-column proofs are not expected at this grade.

What the Teacher does:

- Introduce students to the facts they need to learn in this standard through explorations such as the following: Have each student draw a triangle of his or her choice and tear off the angles. Rearrange the angles so they touch one another. Students should notice that each student was able to create a straight line, thus informally arguing that the three angles of a triangle add up to 180 degrees. Note that formal two-column proofs are not expected at this grade.
- Introduce students to parallel lines cut by a transversal with projection equipment or other technology. Ask students to find pairs of equal angles using any tools they choose. Facilitate a class discussion in which adjacent, vertical, complementary, supplementary, alternate interior, and alternate exterior angles are found.
- Use the example from the standard to tie various facts together in an informal argument.

Misconceptions and Common Errors:

Similarity and congruence are two word students frequently confuse. Help them by creating foldables and advanced organizers such as the Frayer model.

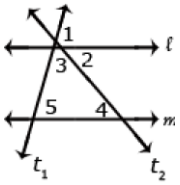
8.G.5. Examples: Students can informally prove relationships with transversals.

Show that m∡3+ *m*∡4+ *m*∡5= 180˚ if *l*and *m* are parallel lines and t_1 & t_2 are transversals.

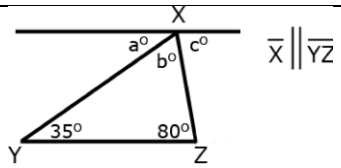
∡1+ ∡2+ ∡3= 180˚. Angle and Angle 5 are congruent because they are corresponding angles (∡5≅∡1). ∡1can be substituted for ∡5.

∡4≅∡2 : because alternate interior angles are congruent. ∡4can be substituted for ∡2

Therefore m∡3+ m∡4+ m∡5= 180˚



Students can informally conclude that the sum of a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line x is parallel to line *yz*:

• Introduce the angle-angle criterion for similar triangles using an exploratory approach. • Pose problems for students to solve using the facts and relationships they have learned in this standard. • Model the use of geometric vocabulary from this standard. Encourage students to create their own dictionary of geometric terms complete with illustrations.	 Angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a + b + c* form a straight line, then angle *b* must be 65 º (180 – 35 + 80 = 65). Therefore, the sum of the angles of the triangle are 35º + 65 º + 80 º **What the Students do:** • Explore angle sums and exterior angles of triangles, angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. Use correct mathematical vocabulary to describe these ideas. • Create and defend informal arguments to justify the facts they have learned. • Solve problems using the facts and relationships studied in this standard such as problems that can be solved by looking at angle relationships on parallel lines cut by a transversal as a model for the problem. **Misconceptions and Common Errors:** Some students have difficulty reasoning with informal arguments. It is best to scaffold questions for those students to help them through an informal argument and supplement that with hands-on materials such as triangles that can be manipulated.
Standards for Mathematical Practice	**Explanations and Examples**
Understand congruence and similarity using physical models, transparencies, or geometry software. **8.G.1, 8.G.2, 8.G.3, 8.G.4, 8.G.5** This cluster focuses on the concepts of congruence and similarity. Students learn about transformations and use them to establish congruence and similarity of figures on a coordinate plane. Triangles and angles are studied by using informal arguments to establish facts about relationships between the angles of triangles and the different types of angles created when parallel lines are cut by a transversal.	
MP3.Construct viable arguments and critique the reasoning of others	Students use informal arguments to establish facts about the angle sum and exterior angle of triangle, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
MP4. Model with mathematics	Students model on the coordinate plane to explore congruent and similar figures.

MP6.Attend to precision	Students are careful to bring lines to lines and angles to appropriate angles in transformations.
MP7. Look for and make use of structure	Students attend to the structure of figures as they transform them.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*

● Vocabulary: See vocabulary section below.
● Transformations include: reflections, translations, rotations, and dilations
● Reflections, rotations, and translations are rigid transformations that maintain congruence
● Congruence and similarity in terms of transformations
● When parallel lines are cut by a transversal, it creates congruency of angles. (Introduced in grade 7 needs to be reviewed.)
● Two triangles are similar if at least two pairs corresponding angles are congruent (AA postulate for similarity)
 	
PRIOR KNOWLEDGE:
● Symbols for congruency and similarity ($\cong$ and ~)
● The sum of the angles of a triangle is 180°.
	● VERIFY experimentally the properties of
	  ○ Rotations
	  ○ Reflections
	  ○ Translations
	  ○ Dilations
	● DESCRIBE a sequence of rotations, reflections, and translations that exhibits congruence between two figures
	● DESCRIBE a sequence of rotations, reflections, translations, and dilations that exhibits similarity between two figures
	● DESCRIBE the effect of dilations, translations, rotations and reflections on two-dimensional figures
	● USE coordinates to describe the effect of dilations, translations, rotations and reflections on two-dimensional figures
	● PROVE informally
	  ○ angle relationships in parallel lines cut by a transversal (review from 7th grade)
	  ○ sum of angles in a triangle = 180°
	● FIND missing angle measures of a triangle
	● FIND missing angle measures when two parallel lines are cut by a transversal
	● DETERMINE if two triangles are similar using the Angle-Angle criterion of similar triangles

UNDERSTAND *Big ideas, generalizations, principles, concepts, ideas that transfer across situations*
Students will understand **that:** ● Spatial relationships help to make sense of the physical space around them. ● A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. ● A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations.
Common Student Misconceptions for this Unit
● Students might believe that ○ A reflection, translation, or rotation creates a similar figure instead of a congruent figure. ○ A dilation creates a congruent figure instead of a similar figure. ○ The x-axis is the y-axis or vice versa. ○ The x-coordinate changes when reflecting over the x-axis or that the y-coordinate changes when reflecting over the y-axis. ○ A clockwise rotation is to the left or that a counterclockwise rotation is to the right. ○ A vertical translation affects the x-coordinate and a horizontal translation affects the y-coordinate. ○ The sum of the angle measures of a triangle is 90° instead of 180°. ○ Complementary angles add to 180° and that supplementary angles add to 90°.

Unit Assessment/Performance Task	**DOK**
Unit 7 Test Part 1 and 2 Unit 7 Performance Task " Aaron's Designs" Unit 7 Performance Task " Playing with Straws"	

Vocabulary
● Alternate interior angles (review from 7th) ● Angle of rotation ● Congruent figures ● Corresponding angles (review from 7th) ● Dilation ● Line of reflection ● Linear Pair (review from 7th) ● Ordered pairs ● Parallel lines

- Reflection
- Rigid motion
- Rotation
- Same-side Exterior Angles (review from 7th)
- Same-side Interior Angles (review from 7th)
- Scale Factor
- Similar Figures
- Similarity transformation
- Transformation
- Translation
- Transversal (review from 7th)
- X axis
- Y axis

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Translations

Students will verify experimentally the properties of translations. They will also describe the effect of translations on two-dimensional figures verbally and by using coordinates.

Give students graph paper and have them follow along with the video as it reviews how to perform a translation on the coordinate plane using a triangle:
http://www.youtube.com/watch?v=XdjH_EWhCZ0

See the following link for Geometer's Sketchpad suggested activity. This activity uses rotations, reflections and translations to create spinning pinwheels. Students can investigate the effect that different angle rotations have on the resulting figure:

http://math.rice.edu/~lanius/misc/index.html

Reflections

Students will verify experimentally the properties of reflections. They will also describe the effect of reflections on two-dimensional figures verbally and by using coordinates.

Use tools such as: mirror boards, patty paper, tracing paper, etc to have students discover the relationships among coordinates when reflecting over the x- and y-axis.

See the following link for Geometer's Sketchpad suggested activity. This activity uses rotations, reflections and translations to create spinning pinwheels. Students can investigate the effect that different angle rotations have on the resulting figure:

http://math.rice.edu/~lanius/misc/index.html

Rotations

Students will verify experimentally the properties of rotations. They will also describe the effect of rotations on two-dimensional figures verbally and by using coordinates.

Lay tracing paper over a coordinate plane with one point plotted in the first quadrant. Have students trace the axes and the point given. Students need to record the coordinates of the original point. Then, rotate the tracing paper 90˚ clockwise around the origin, be sure to line up the axes, and record the coordinates of the new point generated by the rotation. Once the students have rotated the paper back to the origin, have them do a think-pair-share to analyze the pattern of the coordinates after each rotation.

See the following link for Geometer's Sketchpad suggested activity. This activity uses rotations, reflections and translations to create spinning pinwheels. Students can investigate the effect that different angle rotations have on the resulting figure:

http://math.rice.edu/~lanius/misc/index.html

Dilations

Students will verify experimentally the properties of dilations. They will also describe the effect of dilations on two-dimensional figures verbally and by using coordinates.

Given pairs of various figures with their respective dilations and their coordinates, have students identify the pattern between the original and the new coordinates.

Ask students what they know about the word dilation, relate back to the pupils of the eye and their reaction to light—growing and shrinking, yet remaining a circle.

Relate back to seventh grade where they learned that similar figures are proportional to each other and how this applies to a scale factor.

See the following link for an interactive lesson on dilations. Students will be given an opportunity to explore with two given rectangles the effects of changing the scale factor and center point.
http://www.mathopenref.com/dilate.html

Verify that two figures are congruent
Students will be able to describe a sequence of translations, reflections, and rotations that exhibits congruence between two figures.

After students have an understanding of how to reflect, translate, and rotate, have them progress to transforming two-dimensional shapes by recording the steps and then challenge a partner to "reverse" the transformation to obtain the original shape.

Have students work through the following lesson to find the coordinates of a geometric figure after the result of more than one transformation.

Then, have students use their original graph and their new graph to write a verbal description of how the image moved using specific details and coordinates from both graphs. Ask the students "is the original shape the same as the new?"

Verify that two figures are similar
Students will be able to describe a sequence of translations, reflections, rotations, and dilations that exhibits similarity between two figures.

Copy a photograph from a website and paste it into a photo editing program. Have students perform transformations to the original image: enlarge, reduce, translate, rotate, and reflect. Then, have students write each step with a detailed description of the transformation.

Angle relationships of triangles
Students will explore the relationship among the angles of a triangle and find missing measures of a triangle.

Take three congruent triangles and position a vertex from each triangle so that it appears to be a straight line. Give an argument that supports the triangle sum theorem.

Angle-Angle criterion of similar triangles
Students will determine if two triangles are similar using the Angle-Angle criterion of similar triangles.

Give students multiple triangles with various measurements. (Ex. Give some angle measurements and some side length measurements.) Require students to identify which triangles are similar using the angle-angle criterion for similarity. Students should justify their selections.

Parallel lines
Students will examine and identify the relationships of angles formed when two parallel lines are cut by a transversal.

Using manipulatives (spaghetti, scraps of paper, Geometer's Sketchpad, etc.), create two parallel lines intersected by a transversal. Have students label all the angles created and label them. Then, have students use a protractor to measure all the angles created.
Make sure to address all of the following angle relationships:
(Complementary angles – introduced in grade 7)
(Supplementary angles – introduced in grade 7)
Alternate interior angles
Alternate exterior angles
Corresponding angles
Vertical angles
Adjacent angles

Supplemental Materials and Resources

- On Core Mathematics Middle School Grade 8, Houghton Mifflin Harcourt
- https://www.georgiastandards.org/Common-Core/Pages/default.aspx
- http://www.ncpublicschools.org/acre/standards/
- http://www.azed.gov/standards-practices/mathematics-standards/
- http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=322592
- http://www.learnzillion.com

Literature connections:
Alice in Wonderland: Through the Looking Glass

Interdisciplinary connections:

- Quilting
- Biology-pupils dilating
- Architecture/Engineering-using blue prints, scale models, and scale drawings
- Art-creating various drawings using transformations

Tools/Manipulatives
● Patty Paper ● Mirror boards ● Mirrors ● Graph paper ● Compass ● Protractor ● Spaghetti ● Ruler ● Dry erase boards/markers ● Tracing paper ● Geometer's Sketchpad or Geogebra
Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes

Differentiation and Accommodations
● Provide graphic organizers ● Provide additional examples and opportunities for repetition ● Provide tutoring opportunities ● Provide retesting opportunities after remediation (up to teacher and district discretion) ● Teach for mastery not test ● Teaching concepts in different modalities ● Adjust homework assignments