

Grade/Subject	Grade 7/ Mathematics Grade 7/Accelerated Mathematics
Unit Title	Unit 1: Operating with Rational Numbers (Addition/Subtraction)
Overview of Unit	Number System - Apply and extend previous understandings of operations with fractions to add and subtract rational numbers.
Pacing	Grade 7 Mathematics - 23 days Grade 7 Accelerated Mathematics – Units 1 and 2 take 46 days total

Background Information For The Teacher
<p>Students have studied operations with whole numbers, fractions, and decimals in previous grades. In this unit, students should extend this understanding to operations with negative numbers. Exploring ideas about negative numbers by building and connecting to what students already know will not only help develop an understanding of negative numbers, but also deepen their understanding of meaning and operations of positive numbers. Doing this will require students to make meaning of the operations and analyze what kinds of situations call for which operation.</p> <p>Students come to this unit having already informally experienced positive and negative numbers in their everyday lives—temperatures in winter dropping below zero, TV game shows in which participants lose points if they answer incorrectly, and sports teams being ahead or behind by some amount. This unit recommends exploring situations that require students to reason and represent with integers. Number lines offer a wonderful model for developing understanding of order, for comparing integers, as well as for developing the concept of opposites, distances, and absolute value. The number line can also be used to model addition and subtraction.</p> <p>The inverse relationship between addition and subtraction needs to be addressed to help students generalize algorithms for operations as well as looking at number patterns. This is particularly vital later when solving algebraic equations and students need to use the inverse to isolate the variable. Asking questions about meaning and about what makes sense will help focus students’</p>

attention on the situation, the operation and connections.

Essential Questions (and Corresponding Big Ideas)

When am I going to use positive and negative numbers?

- Positive and negative rational numbers help us understand more real world situations.

How are positive and negative numbers related?

- Positive and negative numbers have the same absolute value, which is their distance from zero; whether they're negative and positive is determined by which direction (left or right) the distance moves from zero.

How can I model negatives numbers, including how I operate with them?

- A variety of math tools and real world examples can model negatives, such as number lines, two-color chips, thermometers, underground parking and scenarios such as credits and debits.

How can I use what I already know about adding and subtracting positive numbers to add and subtract with negative numbers?

- The methods for adding and subtracting whole numbers can be applied to adding and subtracting other rational numbers.

How is it possible to add two quantities and get a sum that is less than what you started with?

- Adding two rational numbers does not always result in a greater sum when operating with negative numbers.

Core Content Standards	Explanations and Examples
<p>7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities</p>	<p>7.NS.1 Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Use a number line to illustrate: <ul style="list-style-type: none"> ○ $p - q$

combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

Students use real-world situations that model using opposite quantities to make zero. Thus prepares students for adding rational numbers with opposite signs such as $4 + (-4) = 0$. Examples can include temperature, elevation above and below sea level, owing money, and so on.

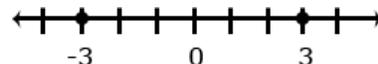
What the teacher does:

- Open a discussion for students to talk about positive and negatives in the real world such as temperature above and below zero, going up and down in an elevator, or owing money.
- Provide students with two color counters (or cubes or tiles), with one color representing positive and the other negative. An equal number of positive and negative counters represents zero. Challenge students to make zero multiple ways. This can also be done with tiles.
- Model on a number line how a certain number of moves in a positive direction from zero combined with the same number of moves in the opposite direction ends at zero on the number line.
- Challenge students to write a real-world story where opposites make zero.
- Provide students with opportunities to write to clarify their understanding of the concept of making zeros. Give a prompt for students to write about in their mathematics journals such as, "What did you learn about positive, negative, and zero? Do you think this is always true? Explain."

b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by

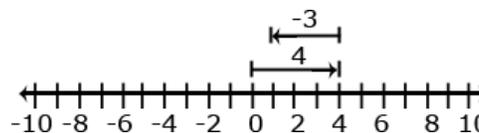
- $p + (-q)$
- Is this equation true $p - q = p + (-q)$

- -3 and 3 are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and it's opposite is zero.



- You have \$4 and you need to pay a friend \$3. What will you have after paying your friend?

$$4 + (-3) = 1 \text{ or } (-3) + 4 = 1$$



What the students do:

- Consider real-world examples in terms as positive and negative such as 30 degrees below 0 is -30.
- Represent multiple "zeroes" by combining the same number of positive and negative counters.
- Model on a number line how a certain number of moves in a positive direction from zero combined with the same number of moves in the opposite direction ends at zero the number line.
- Model positive and negative combining to make zero in real-world situations.
- Communicate understanding of positive, negative, and zero orally and in writing.

Misconceptions and Common Errors:

Students may understand that one positive and one negative make zero but have difficulty understanding that this is also true for all equal amounts of positives and negatives such as five positives and five negatives. One way to make this clear is to start with one positive and one negative counter. As soon as the student establishes that this is zero, add another pair. When the student recognizes that you have just added another zero to the first zero, repeat. Repeat until the student has developed the concept.

describing real-world contexts.

This standard formalized the concept of a positive and a negative making zero from the previous standard into written equations. For example, $4 + (-4) = 0$. The 4 and (-4) are opposites because they are equidistant from 0 on the number line in opposite directions. They are also additive inverses because their sum is 0. Be sure to include examples of fractions and decimals such as $-1/2$ and -4.72 so that students are working with all types of rational numbers. Addition of integers is modeled on a number line as in the following example: "Jose has \$6 and owes Steven \$5. How much money will Jose have left when he pays Steven what he owes?"

What the teacher does:

- Provide students with a mat that has one side labeled "positive" and the other side "negative" and ask students to show zero using their counters (or cubes, tiles, etc.). Define this concept as the additive inverse and ask students to show more examples of additive inverse in the form of $p + q = 0$.
Positive + Negative -
- Challenge students to figure out the sum when five counters are placed on the positive side and four counters on the negative side. Repeat the challenge with different combinations that result in positive and negative sums. Ask students to explain how they arrived at their answers.
- Challenge students to find sums with more combinations of positive and negative integers using number lines. Let students justify their reasoning to classmates. Encourage them to use fractions and decimals also such as $-1/2$ or -4.5 .
- Present equations to students in the form $p + q =$ allow them to solve using number lines, counters, and/or rules they developed.
- Give an equation such as $-17 + 4 =$ and ask students to give a real-world scenario for the equation such as, "I owe my friend \$17. I pay him \$4. How much do I still owe?"
- Encourage a class discussion of any patterns/algorithms/rules students may have developed for adding rational numbers.
- Provide writing prompts such as, "Give a real-world example of additive inverse."

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$.

What the students do:

- Demonstrate an understanding of additive inverse by developing examples.
- Model, using mats, number lines, counters, equations, and so on, different combinations of positive and negative integers and explain how they reason their solutions.
- Solve equations using number lines, counters, and/or rules the students may have developed for addition of rational numbers.
- Discover and apply formal rules for adding rational numbers with different signs.
- Communicate reasoning for addition through writing.

Misconceptions and Common Errors:

Students who are not able to solve equations abstractly as quickly as others may need to use number lines and/or two-color counters for a longer period of time until they understand the concepts. Algebra tiles or blocks may be used as models.

Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Subtraction of rational numbers can be thought of in terms of addition using the additive inverse (sometimes referred to as “the opposite”). For example, $6 - 7$ can be understood as $6 + (-7)$. The distance between two rational numbers on a number line is the same as the absolute value of the difference between the two numbers. For example, using a real-world context, if the temperature is -6 at 7 a.m. and $+8$ at noon, how many degrees has the temperature increased between 7 a.m. and noon? The difference between $-6 - 8 = -14$, $|-14| = 14$. You may show on a number line, the distance between -6 and 8 is 14 .

What the teacher does:

- Provide students with examples of simple subtraction such as $6 - 5$ to model on a number line followed by its corresponding addition with an additive inverse such as $6 + (-5)$. Repeat with progressively more complex examples to model such as $-4.6 - 3$ followed by $4.6 + (-3)$ and $2 \frac{3}{4} - (-1)$ followed by $2 \frac{3}{4} + (1)$.
- Supply students with opportunities to communicate what they are doing and reasoning orally and in writing. Examples can be talking to a partner or writing in math journals.
- Provide repeated opportunities for students to subtract. During those practices, encourage students to discover a formal rule for subtraction of rational numbers during large class discussion so that students can justify their rules to one another.
- Engage students with real-world context to demonstrate the distance between two rational numbers on the number line as the absolute value of their difference such as, “How far did the temperature rise on Monday if the low temperature was -1 and the high was 3 ?”

d. Apply properties of operations as strategies to add and subtract rational numbers.

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Students have previously used the commutative, associative, and additive identity properties with whole numbers. These properties apply to rational numbers. For example:

Commutative Property of Addition:
 $4.5 + (-6) = (-6) + 4.5$

Associative Property of Addition:
 $6.9 + (-5) + 3.2 = 6.2 + 3.9 + (-5)$

Additive Identity Property of Addition (also called the Zero Property):
 $(-4.8) + 0 = (-4.8)$

What the teacher does:

- Provide students with examples and non-examples of commutativity, associativity, and additive identity of addition with rational numbers for students to use to clarify their understanding of the properties.
- Use an advance organizer such as the Frayer vocabulary model to help students clarify their understanding of the properties as they apply to rational number operations.
- Provide students with numerical equations to solve using the properties of the operations, such as; $-1/2 + 2.4 + 1/2 = (-1/2 + 1/2) + 2.4 = 0 + 2.4 = 2.4$.

7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Extend the work with order of operations to all rational numbers. An example of a mathematical problem is $-3 \times 2(\frac{5}{6} + -\frac{1}{2}) = -2$.

Complex fractions are fractions with a fraction in the numerator

and/or a fraction in the denominator such as $\frac{\frac{3}{4}}{\frac{1}{2}}$. Interpret the division

bar to turn a complex fraction into division:

$$\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \div \frac{1}{2}$$

What the students do:

- Discover that subtraction and adding with an additive inverse provide the same results. For example, $(-1/2) - 5$ is the same as $(-1/2) + (-5)$.
- Clarify their reasoning about subtraction and additive inverse through oral and/or written communication.
- Model real-world contexts that involve subtraction of rational numbers using a number line.
- Discover that the solutions to real-world subtraction problems using the absolute value of the distance between two rational numbers on the number line give the same result as subtracting through examples given in class.

Misconceptions and Common Errors:

For students having difficulty understanding subtraction as adding the inverse using a number line, use the mats and two color chips. Demonstrate the equation $5 - 6 = (-1)$ on mat. First place 5 positive chips on the mat. Then, try to remove 6. Since this is not possible, add a zero as a pair of chips.

Now it is possible to remove 6 chips (subtract 6). You are left with one chip on the negative side of the map. This mat exercise can be used to model any subtraction.

Mathematics/Grade7 Unit 1: Operating with Rational Numbers (Addition/Subtraction)

<p><u>What the teacher does:</u></p> <ul style="list-style-type: none">• Extend the work with order of operations to all rational numbers.• Provide real-world problems that build on previously studied skills. For example: “You want to buy a new tablet. The service agreement will deduct \$22.50 from your savings every month to pay for it. How much will the deductions be at the end of the year?” $12(-22.50) = -270$. Include problems that apply all four operations with rational numbers and complex fractions.	<p><u>What the students do:</u></p> <ul style="list-style-type: none">• Discover that properties of operations apply to addition and subtraction of rational numbers by identifying examples and non-examples.• Clarify their understanding of the properties of operations as they apply to addition and subtraction of rational numbers by completing an advanced organizer.• Solve numerical addition and subtraction equations using the properties of the operations. <p><u>Misconceptions and Common Errors:</u></p> <p>In previous grades students learned that subtraction is not commutative. This holds true with rational numbers even though students now understand that $6 - 8 = (-2)$. It is still the case that $6 - 8$ does not equal $8 - 6$.</p> <p>7.NS.3 Examples:</p> <ul style="list-style-type: none">• Your cell phone bill is automatically deducting \$32 from your bank account every month. How much will the deductions total for the year? $-32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12 (-32)$• It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? $\frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5 \text{ ft/sec}$ <p><u>What the students do:</u></p> <ul style="list-style-type: none">• Apply operations with rational numbers to problems that involve the order of operations.• Solve mathematical problems that use the four operations with rational numbers.• Solve real-world problems that involve the four operations with rational numbers.• Compute with complex fractions.
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Mathematics/Grade7 Unit 1: Operating with Rational Numbers (Addition/Subtraction)

	<p><u>Misconceptions and Common Errors:</u></p> <p>As equations become longer with more terms and more complex using rational numbers, some students are overwhelmed and do not know where to begin. Help these students by reviewing the order of operations and demonstrating how to solve equations one step at a time. Flip books created by students that do a step-by-step breakdown of a computation aid some students. For such a book, students can begin with a problem and perform one step on the first page, then repeat that step and add a second step to next page, continuing in this manner.</p>
<p>Standards for Mathematical Practice</p>	<p>Explanations and Examples</p>
<p>Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divided rational numbers. 7.NS.1, 7.NS.3</p> <p>This cluster is about understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Students learn how to add, subtract, multiply, and divided integers and apply properties of operations as strategies for each operation. Students journey from exploring the operations to formalizing rules. Students convert rational numbers to decimal form using division. The understanding of a rational number as one that terminates or repeats is covered in Grade 7 as preparation for the introduction of irrational numbers in Grade 8.</p> <p>MP4. Model with mathematics.</p> <p>MP6. Attend to precision.</p> <p>MP7. Look for and make use of structure.</p> <p>MP8. Look for and express regularity in repeated reasoning.</p>	<p>Students use multiple strategies to demonstrate the same meaning of an operation which include modeling with manipulatives or a on a number line.</p> <p>Students are working towards being independent thinkers by self-correcting any errors they may find.</p> <p>Students make use of what they already know about operations and their properties and extend the understanding rational numbers.</p> <p>Students use examples of integer multiplication to generalize a general rule.</p>

K-U-D	
KNOW <i>Facts, formulas, information, vocabulary</i>	DO <i>Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)</i>
<ul style="list-style-type: none"> • Strategies to represent and solve problems involving operations with rational numbers (including decimals, fractions, integers) • The sum of two opposites is zero • A negative number can also be interpreted as the opposite of the positive number. (Ex: -5 can be interpreted as the opposite of 5.) • Absolute value of a rational number is its distance from zero on a number line • Computation with integers is an extension of computation with fractions and decimals. • Strategies utilized for adding & subtracting fractions and decimals numbers extend to integers. Ex: The same reasoning used to solve $6 + 2$ can be used to solve problems involving integers such as $6 + (-2)$ • Models: Number line, chip model, area model, arrays, bar model, fraction circles, picture/visual 	<ul style="list-style-type: none"> • LOCATE rational numbers on a number line. • COMPARE numbers on a number line. • ADD and SUBTRACT rational numbers • DESCRIBE the relationship of opposite quantities • UNDERSTAND positive or negative direction on a number line • SHOW additive inverses • INTERPRET sums in context • SHOW absolute value • APPLY absolute value principle in context • APPLY properties of operations as strategies • SOLVE addition & subtraction equations involving rational numbers with and without context <ul style="list-style-type: none"> ○ APPLY properties of operations to calculate ○ CONVERT between equivalent forms ○ USE mental computation and estimation strategies to ASSESS reasonableness of answers • REPRESENT addition and subtraction on a horizontal or vertical number line • APPLY operations of rational numbers to real-world problems • WRITE rational numbers as terminating or repeating decimals using long division.
UNDERSTAND <i>Big ideas, generalizations, principles, concepts, ideas that transfer across situations</i>	
<ul style="list-style-type: none"> • Rational numbers are a subset of the number system, including and beyond whole numbers. 	

- Properties of whole number operations can be applied to solving real world and mathematical problems involving rational numbers, including integers.
- Subtraction of a rational number is thought of as adding an opposite because the negative sign reverses your direction on the number line, $p - q = p + (-q)$.
- Estimation is a means for predicting & assessing the reasonableness of a solution.

Common Student Misconceptions for this Unit

- When I add two numbers, my answer is always greater than the two numbers that were added.
- When I subtract two numbers, my answer is always less than the two numbers that were subtracted.
- It is not possible to subtract a larger number from a smaller number.
- When faced with a double negative situation, students change one of the negative signs but not the other. Example: $3 - (-9) = 3 - 9$ or $3 - (-9) = 3 + (-9)$

Unit Assessment/Performance Task	DOK
Unit 1 Test will be given as a dual assessment in Unit 2 Unit 1 Performance Task "Rational Numbers (addition and subtraction)	

Vocabulary

- Absolute Value Opposites
- Additive Inverse
- Complex Fraction
- Integers
- Negative fractions
- Number Line
- Opposite Quantities
- Rational Numbers

- Repeating Decimal
- Terminating Decimal

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons. Teachers can refer to Math In Focus Chapters 1 Sections 1.1 and 1.2 and Chapter 2 Sections 2.1, 2.2, 2.5, 2.6

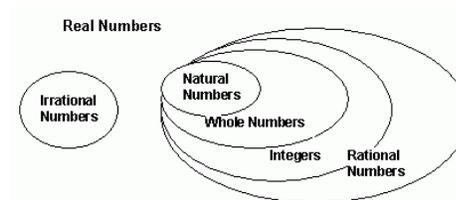
Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

This unit does not develop the arithmetic of integers before the arithmetic of rational numbers in general. Rather, each specific phase of the outline would likely begin with integers for simplicity and incorporate other rational numbers second.

Lesson 1: Comparing and Ordering Rational Numbers

Goals

- Students will understand the value of rational numbers.
- Students will understand that opposite quantities combine to make zero.
- Students will understand situations that real-world problem solving requires an understanding of positive and negative rational numbers.



Suggested Lesson Activities:

- Introduce the term ‘Rational Numbers’
- Show and label image to the right to include examples of numbers in each category.
- As a class and then in pairs, use number lines (horizontal and vertical) to compare and order positive and negative rational numbers including fractions, decimals and zero so that students understand the value of rational numbers. When plotting fractions and decimals students may need reminders to divide the space between whole numbers into equal parts.
- Have students prove that opposites are numbers the same distance from zero but in opposite directions on a number line so that students will understand that opposite quantities combine to make zero. Describe situations such as earning \$5 then spending \$5, putting 3 apples in a

basket then removing 3 apples from the basket and have students come up with several situations on their own.

- Assign students pairs of numbers that are examples of and not examples of absolute value pairs and have them construct viable arguments to prove or disprove using words or pictures. This will help students to later understand the rules for adding integers.
- Introduce students to the real-world uses of integers so they realize that some situations require an understanding of positive and negative rational numbers.

Lesson 2: Adding Rational Numbers with the Same Signs

Goals

- Students will use models and rules to add integers with the same signs.
- Students will extend their understanding of adding integers to include fractions and decimals.

Suggested Lesson Activities:

- Use both horizontal and vertical number lines to show addition of integers. Make connections to real-life situations such as the temperature dropping 12 degrees and then 4 more degrees. Remember to also include fractions and decimals in your examples, such as riding a bike 2.5 miles and then 3.25 more miles,
- Give pairs of students' number lines along with situations to represent. They can also write equations to represent each situation.
- Give pairs of students yellow (positive) and red (negative) color counters. (small plus signs and minus signs might be helpful for struggling students.) Tell them to use the chips to model problems you give them where they will have to add numbers with the same signs. ($3 + 4 = ?$, $-6 + -2 = ?$)
- Tell students to work with their partner to write a rule to explain how to add integers with the same signs.
- Come back together as a group to review student rules and develop one common rule.
- Introduce students to the real-world uses of integer addition so they realize that some situations require an understanding of positive and negative rational numbers.

Lesson 3: Adding Rational Numbers with Different Signs

Goals

- Students will use models and rules to add integers with different signs.
- Students will extend their understanding of adding integers to include fractions and decimals.

Suggested Lesson Activities:

- Use both horizontal and vertical number lines to show addition of integers with different signs. Make connections to real-life situations such as the temperature dropping 12 degrees and then rising 5 degrees. Remember to also include fractions and decimals in your examples, such as driving a car 4 1/2 miles north then 1 1/4 mile south.
- Give pairs of students number lines along with situations to represent. They can also write equations to represent each situation.
- Tell students to separate the counters in several groups that would each show a zero value in order to reinforce previous learning of opposites. (For example, 3 reds and 3 yellows, 5 reds and 5 yellows)
- Ask student to take one of the groups they made and discard the rest. Reminding them that the value of the group in front of them currently equals zero, tell them to add one yellow chip to the group. Ask them what the value of the group is now. Can they write an addition equation to show what they have in front of them now? Do this several times, asking them to add different amount of red or yellow chips to their original group, and write equations for each.
- Give students several mathematical sentences and tell them to use their chips to model each one and then get an answer. As they get more comfortable tell them to do some problems without the chips.
- Ask if they see any patterns in the answers to their equations. You should look for responses such as, “If the positive number is higher then the answer is positive” and vice versa, and “the answer is the difference between the two numbers.”
- Tell students to work with their partner to write a rule to explain how to add integers with different signs.
- Come back together as a group to review student rules and develop one common rule.

Lesson 4: Subtracting Rational Numbers

Goals

- Students will use models and rules to subtract integers.
- Students will extend their understanding of subtracting integers to include fractions and decimals.

Suggested Lesson Activities:

- Use color counter to model subtracting integers. (see teachingchannel.org)
- Begin with simple principles. Remind students of their previous study of subtraction:
 - Check answers to subtraction by adding. We check our answer to $a - b = c$ by adding $b +$

c and making sure we get back to a . This is still true for rational numbers including integers. If ever in doubt about a subtraction, we check our results by adding.

- Subtracting anything from itself gives zero: $a - a = 0$. This would probably just strike most students as common sense. It is true because if you add $a + 0$ you get back to a .
- Subtracting zero leaves a number unchanged: $x - 0 = x$. Again plausible, and true because if you add $x + 0$ you get back to x . The teacher does a few numerical examples at the board to make sure people are good with this. The presentation includes contextual examples in addition to the above mathematical reasoning.

- **Subtracting means adding the opposite.** During their work with addition, students likely already noticed special cases of the general rule. For example, in $(+10) + (-3)$, the answer $+7$ was like subtracting 3 from 10. Students collect these suggestive examples and observe that subtractions familiar from previous grades can be recast as adding the opposite. For example, $4/5 - 1/5 = 3/5$ is familiar from previous grades, and more recently one may recall the result of $4/5 + (-1/5)$ as being $3/5$ as well. So in this case, $4/5 - 1/5 = 4/5 + (-1/5)$. From such examples, supported as appropriate by models and contextual situations, students conjecture, or are led to conjecture, that all subtraction of rational numbers is adding the opposite. Students make a small test of this conjecture by showing how this rule agrees what came immediately before: namely, it reproduces the prior conclusions that $a - a = 0$ (because $a + (-a) = 0$) and that $x - 0 = x$ (because $x + (-0) = x + 0 = x$).
- Students should practice subtraction of two rational numbers, including fractions and decimals. In some cases they should be asked to check the results by addition.

Lesson 5: Working with parentheses

Goals

- Students will extend their knowledge of addition and subtraction with rational numbers to more complex problems.

Suggested Lesson Activities:

- Students work with **parentheses**. The first problem they do is $-(8+5) = ?$. This is easy if we begin inside the parentheses, $8+5 = 13$ so $-(8+5) = -13$. Students are shown this result in a new light: because $-13 = -8 + -5$, our result can be written as $-(8+5) = -8 + -5$. This can be described by saying that **“the opposite of a sum is the sum of the opposites.”** Students might be invited to think of this as “distributing in the minus sign.” (The use of the term “distributing” will be

justified later during the study of multiplication of rational numbers).

- Next problem: $6 - (8+5) = ?$ We can do this as $6 - (8+5) = 6 - 13 = 6 + (-13) = -7$. In that approach, we evaluate parentheses first as usual and then apply the principle that subtraction is adding the opposite. Or, we can view the term in parentheses as a “chunk” and apply the subtraction principle first: $6 - (8+5) = 6 + -(8+5)$. As a second step, we can evaluate the term in parentheses; this leads to $6 + (-13)$ as before. Alternatively, as a second step we can handle the term in parentheses by applying the principle that the opposite of the sum is the sum of the opposites:

$$6 + -(8+5) = 6 + (-8) + (-5) .$$

- Students should practice by evaluating simple numerical expressions with sums and differences of positive and negative fraction and decimal numbers that involve parentheses and then move to more complex problems with parentheses, and problems with more than one subtraction operation. The goal is fluency.

Lesson 6: Applying the Principles

Goals

- Students will apply the subtraction principle to solve word problems involving the distance between two numbers.

Suggested Lesson Activities:

- Students will work on problems such as: a weather balloon is 100,000 feet above sea level, and a submarine is 3 miles below sea level. How high above the submarine is the weather balloon? John was \$3.75 in debt, and Mary was \$0.50 ahead. John found an envelope with some money in it, and after that he had the same amount of money as Mary. How much was in the envelope? On their way to giving the answer, students should represent these problems by a math diagram, a number line, and an equation.

Further developments during Grade 7

From this point on, algebra work with expressions and equations should sometimes involve positive and negative rational numbers, including positive and negative fractions and decimals, as well as parentheses. Also, applied percentage problems should sometimes involve negative percent differences.

Supplemental Materials and Resources

Literature connection:

- Newspapers
 - Weather (temperature, precipitation, pressure, etc.)
 - Stocks
 - Catalogs/Flyers
- Shopping on a budget

Interdisciplinary connections:

Science

- The balance of protons and electrons in an atom
- Chemical bonds (balance of charges within them)
- Elevations above and below sea level
- Input/output systems
 - Diffusion through cell membranes
- Changes in temperature (above and below 0)
- Motion/Velocity/Acceleration (movement forward and backward in relation to a specified reference point)
- Changes in volume/mass/density

Social Studies/Geography

- Increases and decreases in populations
- Elevations above and below sea level

National Library of Virtual Manipulatives

<http://nlvm.usu.edu/en/nav/vlibrary.html>

Online Lessons

- Adding and Subtracting Integers (Lesson with worksheets)
<http://www.uen.org/Lessonplan/preview.cgi?LPid=23473>
- Adding and Subtracting Integers Using a Number Line (Lesson with worksheets)
<http://www.nylearns.org/module/content/pyb/resources/13166/view.ashx>
- Using Positive and Negative Numbers in Context (Lesson with student materials)

<http://map.mathshell.org/materials/download.php?fileid=1304>

Worksheets

- Rational Numbers (Intro) & Compare and Order Rational Numbers Lessons and Practice
<http://teachersites.schoolworld.com/webpages/TAhrens/files/Rational%20Numbers.pdf>
- Adding and Subtracting Integers
<http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/2-1/GlencoePWS2-3.pdf>
- Adding and Subtracting Integers Study Guide (*Note: Use this worksheet only AFTER students have had sufficient practice with adding and subtracting integers to discover the rules for themselves!*)
<http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/2-1/GlencoeSG2-3.pdf>

Videos

- Finding Absolute Value (Pearson)
http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-145s.html
- Adding and Subtracting Integers (BrainPOP)
<http://www.glencoe.com/sec/math/brainpops/00112034/00112034.html>
- Zero Pairs, Manipulatives, and a Real-World Scenario (from TeachingChannel.org)
<https://www.teachingchannel.org/videos/teaching-subtracting-integers?fd=1>
- What's Your Sign: Integer Addition (from TeachingChannel.org)
<https://www.teachingchannel.org/videos/adding-integers-lesson-idea?fd=1>
- Adding Integers (from TeachingChannel.org)
<https://www.teachingchannel.org/videos/math-teaching-techniques?fd=1>

SMART Board Lessons

- Integers – Students will be able to learn how to use integers with addition, subtraction, multiplication and division.
<http://exchange.smarttech.com/details.html?id=6ff222a2-413a-4832-9859-4f17e82389a7>
- Addition, Subtraction, Multiplication, and Division of Integers – This lesson activity provides practice of the rules for the basic operations with integers.
<http://exchange.smarttech.com/details.html?id=fc8465eb-246c-4c6f-9153-d0c3b681ed30>
- And many more: [HERE!](#)

Online Interactive Activities & Games
<ul style="list-style-type: none">• Adding Integers using an Interactive Number Line• Subtracting Integers using an Interactive Number Line• Adding and Subtracting Integers Online Test• Circle 0 – A puzzle involving adding positive and negative integers to sum to zero.• Color Chips – Subtraction – Use color chips to illustrate subtraction of integers.• Integer Games
Tools/Manipulatives
<ul style="list-style-type: none">• Number Lines<ul style="list-style-type: none">○ Create custom number line worksheets: http://www.math-aids.com/Number_Lines/Number_Lines.html• Color Tiles/Counters• Playing Cards• Dice

Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes Think-Pair-Share Agreement Circles Fist to Five: Have students self-assess how well they understand the concept with a show of fingers (from 0-5, 0 = no understanding, up to 5, which = a complete understanding)

Differentiation and Accommodations

- Provide graphic organizers
- Provide additional examples and opportunities for repetition
- Provide tutoring opportunities
- Provide retesting opportunities after remediation (up to teacher and district discretion)
- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments