

Grade/Subject	Grade 7/ Mathematics Grade 7/Accelerated Mathematics
Unit Title	Unit 6: Probability
Overview of Unit	In this unit, students will determine the probability of chance events (simple and compound) through experimentation with concrete materials. They will develop probability models for both simple and compound events, compare experimental and theoretical probabilities, and use information from simulations for predictions.
Pacing	Grade 7 Mathematics – Grade 7 Accelerated Mathematics -

Background Information For The Teacher

Rationale

To make sense of the concepts involved in probability, 7th grade students need to conduct a variety of experiments, collect and analyze data, and make inferences about the chances of relatively *simple* events using concrete materials (dice, spinners, coins, cards, etc.) and methods of analysis (organized lists, tree diagrams, etc.). Students will then move to more *complex* experiments (compound events) and be able to use these methods of analysis across a variety of contexts. At the end of the unit, students will demonstrate their knowledge and understanding of probability by completing a performance task where they will create a carnival game and calculate and compare its corresponding probabilities.

Key Learning

Students discover in this unit that they can use probability to make predictions and will come to the realization that it may not always unfold in real life that way. However, they will learn that simulations of real-world situations can be used to predict the likelihood of future outcomes. They will also discover that chance has no memory and that for repeated trials of a simple experiment, the outcome of prior trials has no impact on the next. Students will determine that there are some decisions, such as investments, finding a job, winning the lottery, college acceptance, etc., that can be influenced and better informed by probability calculations.

Displaced Concepts

Probability is no longer taught in any previous grade!

Probability is not included in 8th grade Standards.

More formal reasoning and application of probability concepts do not occur until High School Geometry (Unit 9: Applications of Probability).

Changes from Past Practice

The concept of probability has been displaced from all previous grades. Teachers will need to spend several weeks on the concept to help students develop full understanding of both theoretical and experimental probabilities. In addition, this unit will provide a context for developing standards for mathematical practice #7: look for and make use of structure and #8: look for and express regularity in repeated reasoning, which were not as deliberately articulated in previous curricula.

Essential Vocabulary for the Teacher (if any)

- **Uniform probability** - A probability model, which assigns the same (equal) probability to all outcomes. (ex: a spinner that has 4 equal sections)
- **Compound events** – Probabilities of two or more things happening at once.
- **Relative frequency** – The ratio of the actual number of favorable events to the total possible number of events; often taken as an estimate of probability
- **Long-run relative frequency** – The probability of an event when an experiment is carried out many times.

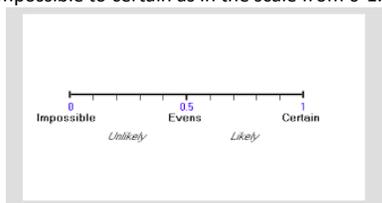
Possible Teacher Misconceptions

- In standard 7.SP.6, it states, “... predict the approximate relative frequency given the probability.” Teachers should focus first on the concept of approximation and not on the procedure in which it can be solved. Automatically teaching students to take the probability and multiply it by the number of events to occur is a procedure that, if taught first, will undermine conceptual understanding. We want students to eventually come to this conclusion on their own without being told.

Essential Questions (and Corresponding Big Ideas)	
<ul style="list-style-type: none"> • How can past events help to predict future events? • When/how do we take chances? • How can the circumstances surrounding a situation be adjusted in order to produce a more desirable outcome? 	
Core Content Standards	Explanations and Examples

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

This standard introduces students to the concept of chance with events that are likely, unlikely, or neither likely nor unlikely. Students learn to use a scale from 0-1 representing probabilities that range from impossible to certain as in the scale from 0-1.



Numerical probabilities are numbers from 0-1, and the larger the number (the closer to 1), the more likely the event is to occur. A number near 0 (i.e., $\frac{1}{50}$) indicates an unlikely event and a number in the middle (0.5) is neither likely nor unlikely. A 0 probability is an impossible event, and a 1 is certainty. Probabilities are expressed as ratios of the number of times an event occurs to the total number of trials performed. Probabilities can be represented as a fractions, decimals, and percents.

What the teacher does:

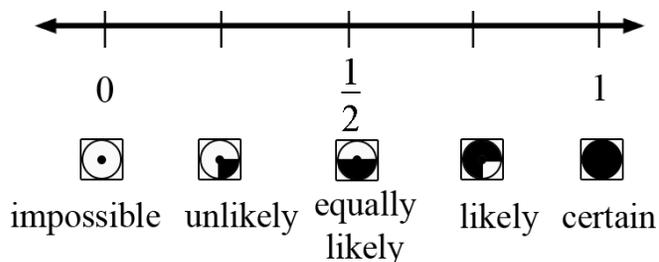
- Promote discussion about events in the students' lives and how likely they are to occur. For example, "How likely is it that lunch will be served today at 3:00?"
- Display a scale from 0-1 and have students add the events they discussed previously to the scale based on their chance of occurring.

7.SP.5 Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line. Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

Marble Mania

<http://www.sciencenetlinks.com/interactives/marble/marblemania.html>

Random Drawing Tool - <http://illuminations.nctm.org/activitydetail.aspx?id=67>



Example:

- The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.



What the students do:

- Understand that probabilities are numbers from 0-1 that express the likelihood of the event occurring. That

<ul style="list-style-type: none"> Present students with blank scales and have them add daily events. Give students specific events to categorize as unlikely, likely, or neither likely nor unlikely. Present students with a list of events and their probabilities and ask students to justify them. Include some probabilities over 1 to be certain students understand the 0-1 scale and that anything over 1 is not a probability. Conduct simple probability experiments. Examples include tossing dice, flipping coins, marbles in a bag, and so on. Use these calculations (expressed in fraction, decimal, and percent forms) to determine if the events are likely, unlikely, or neither likely nor unlikely. 	<ul style="list-style-type: none"> probability closer to 1 are likely, and those closer to 0 are unlikely. Use reasoning to determine where a probability lies on the scale when the probability is expressed as a fraction such as $\frac{5}{8}$. Understand that the probability of 1 is certain and 0 is impossible. Justify the categorization of events as likely, unlikely, or neither likely nor unlikely and use appropriate vocabulary and the concept of probability being from 0-1. Conduct simple experiments and calculate probabilities. <p><u>Misconceptions and Common Errors:</u></p> <p>Students who do not have a solid concept of fractions will have difficulty deciding if a probability is closer to 1 or 0. Encourage these students to convert fractions to decimals or percents.</p> <p>Some students may decide that selecting a blue marble from a bag with 2 blue marbles and 3 red has a probability of 2 instead of $\frac{2}{5}$. Refer these students back to the scale that only goes from 0-1.</p>
<p>7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i></p> <p>Students collect data on chance events so that they can estimate the probability of the event. Students learn the difference between theoretical probability (probability that is calculated mathematically) and experimental probability (actual outcomes of an experiment). Seldom are the theoretical and experimental probabilities equal, although the more a simulation is repeated, the closer the theoretical and experimental probabilities become.</p> <p>Relative frequency is the observed number of successful outcomes in a set number of trials. It is the observed proportion of successful events. Students learn to make predictions about relative frequency of an event by using simulations.</p> <p><u>What the teacher does:</u></p> <ul style="list-style-type: none"> Conduct a simple experiment with a large number of trials (i.e. spinning a spinner, marbles in a bag, coin toss). Let the 	<p>7.SP.6 Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.</p> <p>Example:</p> <p>Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?).</p> <p>Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.)</p> <p>Students try the experiment and compare their predictions to the experimental</p>

<p>students determine the theoretical probability first, and then collect data individually and compile class data. Use these data to introduce the terms theoretical and experimental probability.</p> <ul style="list-style-type: none"> • Simulate chance events in class with students collecting the data using physical objects (marbles in a bag, spinners, etc.). Students can perform experiments multiples times and pool data with other groups to look at the long-run relative frequencies. • Have students observe the long-run relative frequency of the outcomes and predict the approximate relative frequency given the theoretical probability. For example when rolling a number cube 600 times, predict that a 3 or 6 should be rolled 200 times (experimental probability) but probably not exactly 200 times. • Introduce students to the use of simulations with technology to collect data on chance events. Students can collect data using graphing calculators or computer simulations found on the web (random number generators are an example). 	<p>outcomes to continue to explore and refine conjectures about theoretical probability.</p> <p><u>What the students do:</u></p> <ul style="list-style-type: none"> • Explain the difference between experimental and theoretical probability using appropriate vocabulary and examples. • Collect data on chance events (hands-on events such as spinning a spinner and simulations) and approximate the relative frequency of an event given the probability. <p><u>Misconceptions and Common Errors:</u></p> <p>Students may want to express the relative frequency as a probability. Point out that the probability helps to determine the approximate relative frequency.</p>
<p>7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p>a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i></p> <p>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <i>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do</i></p>	

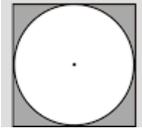
the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Standard 7.SP.7 is broken into two parts (7.SP.7 a-b). We will consider them together since they are so closely related.

Overall, students develop and use probability models to find the probability of events. Uniform probability models are those where the likelihood of each outcome is equal. For example, there are 17 children in the class. What is the probability that Sam will be chosen? Using theoretical probability, students can predict frequencies of outcomes. In part b of this standard, students look at the experimental probability to develop a model.

What the teacher does:

- Present numerous opportunities for students to find the theoretical probability of an event. Have students compare their theoretical probabilities to the experimental probabilities (observed frequencies). Students should be guided to conclude that the number of trials affects the closeness of the experimental probability to the theoretical. Experiments can be hands-on or use a variety of random generation devices, including spinners, number cubes, coin tosses, and colored chips. Students can collect simulation data using graphing calculators or computers for web based simulations. Students can also develop models for geometric probability such as the following: The square has a side length of 16. If you choose a point in the square, what is the probability that it is not in the circle?



- Set-up probability experiments for students to find the approximate probabilities from observed frequencies. Ask questions such as, "Do the outcomes appear to be equally likely? Why do you think the experimental probability was not close to the theoretical in this case? What could cause the discrepancy? What could we do to get different results?"
- Provide students many opportunities to explain their thinking aloud and/or in writing.

<p>opportunity to find the similarities and differences between the two types of events, stressing that the probability of a compound event is the fraction of outcomes in the samples space for which the compound event occurs, just as it is for single events.</p> <ul style="list-style-type: none"> • Provide students opportunities to express their understanding of compound events orally and in writing using appropriate vocabulary: compound event, single event, sample space, outcomes, journals, and entrance and exit slips are some examples of opportunities. • Give students many opportunities to read/answer questions from sample spaces, and create sample spaces for compound events using organize lists, tables, and tree diagrams. Students identify the outcomes. The following example shows a tree diagram: What is the probability of rolling an even number and heads on a coin toss? The answer is $\frac{3}{12}$ or $\frac{1}{4}$. Highlights in the diagram indicate the outcomes in the sample space that make up an event. • Provide many opportunities for students to use simulations to collect data on compound events. Once students are comfortable with simulations, have them design their own to model a compound event and generate frequencies (data) so that students can approximate probabilities for their event. Example of student-design simulations are using a graphing calculator to generate random numbers to represent certain random characteristics of a populations such as the question posed in the standard about blood types. <p>When using tree diagrams students may have difficulty keeping the lines straight and cannot read their final product. Encourage students to use graph paper so they can keep the outcomes apart from each other and/or use a ruler to read across the diagram. When students create lists randomly, there is a greater chance that they will miss listing one or more outcomes.</p>	<p><u>What the students do:</u></p> <ul style="list-style-type: none"> • Explain orally and in writing the similarities and differences between single and compound events. • Read and create sample spaces as organized lists, tables, or tree diagrams to determine the probability of a compound event. • Select the appropriate tools for a simulation for a compound event and use the data it generates to approximate the probability of an event. Explain orally and/or in writing how the simulation was selected, why is models a compound event and not a single event, the data it generated, and how the probability was approximated. <p><u>Misconceptions and Common Errors:</u></p> <p>When using a tree diagram students may have difficulty keeping the lines straight and cannot read their final product. Encourage students to use graph paper so they can keep the outcomes apart from each other and/or use a ruler to read across the diagram. When using lists, some students still need additional help keeping them organized rather than random. When students create lists randomly, there is a greater chance that they will miss listing one or more outcomes.</p>
<p>Standards for Mathematical Practice</p>	<p>Explanations and Examples</p>
<p>Investigate chance processes and develop, use, and evaluate probability models. 7.SP.5, 7.SP.6, 7.SP.7, and 7.SP.8 This cluster focuses on probability and is the first time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.</p>	

<p>MP2. Reason abstractly and quantitatively.</p> <p>MP4. Model with mathematics</p> <p>MP5. Use appropriate tools strategically.</p> <p>MP6. Attend to precision.</p>	<p>Students use reasoning to determine the likelihood of an event.</p> <p>Students construct and use probability models for chance events.</p> <p>Students select from tree diagrams, organized lists, tables, and simulations to determine probabilities.</p> <p>Students calculate probabilities.</p>
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K-U-D	
KNOW <i>Facts, formulas, information, vocabulary</i>	DO <i>Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)</i>
<ul style="list-style-type: none"> • Probability gives a quantitative description of the likelihood of an event. • Probability ranges from 0 to 1. • Probability can be represented as a fraction, decimal, or percentage. • Probability notation • Theoretical Probability • Experimental Probability • Compound events • Sample (Space, size, etc) • Representations: <ul style="list-style-type: none"> - Organized lists - Tables - Tree diagrams • Simulation 	<ul style="list-style-type: none"> • Determine the likelihood of an event. • Approximate the probability of a chance event • Collect data on chance processes of events • Observe long-run relative frequency. • Predict the approximate relative frequency given the probability. • Develop a probability model. • Find probabilities of events (simple and compound). • Use organized lists, tables, and tree diagrams to represent sample spaces. • Create simulations to represent simple events. • Create simulations to represent compound events. • Compare/contrast various types of probability. • Explain possible inconsistencies between modeled and observed frequencies. • Alter elements and/or circumstances to produce a more favorable outcome.

UNDERSTAND
<i>Big ideas, generalizations, principles, concepts, ideas that transfer across situations</i>
<p>Students will understand that:</p> <ul style="list-style-type: none"> • Probabilities lead to predictions, not certainties. • Probabilities can be derived from modeling real world experiments and simulations of chance. • Chance has no memory. For repeated trials of a simple experiment, the outcome of prior trials has no impact on the next. • Probability calculations can be used to make informed and influential decisions, which may occur in a variety of real world contexts.
Common Student Misconceptions for this Unit
<ul style="list-style-type: none"> • Students might believe that the probability of independent events will change based on previous outcomes. For example, if a coin is tossed 10 times and the first 5 times it lands on tails, they then expect it to land on heads 5 times as well since the theoretical probability of landing on either heads or tails is $\frac{1}{2}$. • Students might believe that the probability of an outcome can be based solely on the small number of experimental outcomes. The experimental probability will not always replicate the theoretical probability of an event but as the number of trials increases, the experimental probability will get closer and closer to the theoretical probability. For example, the theoretical probability of getting tails when flipping a coin is $\frac{1}{2}$. If you flip a coin 10 times, you might only get tails 3/10 of the time. As you increase the number of trials, the probability of tails will begin to more closely approximate the theoretical probability of $\frac{1}{2}$. • Students might believe that a game itself is “fair” even though the probabilities for each outcome are not. For example, if 4 people are playing a game and use a spinner that is not divided into 4 equal parts but each of the 4 people win the same number of times, students mistakenly believe that the game itself is “fair” even though the probabilities for each outcome are not $\frac{1}{4}$.

Unit Assessment/Performance Task	DOK
Unit 6 Test Unit 6 Performance Task "Counters"	

Vocabulary
<p><u>Academic Vocabulary</u></p> <p>Approximate -To come close to or be similar to something in quality, nature, or quantity</p> <p>Certain - Known for sure; established beyond a doubt</p> <p>Collect - Gather information</p> <p>Compound Event - Two or more independent events considered together</p> <p>Data - Facts and statistics collected together for reference or analysis</p> <p>Experiment - A procedure undertaken to make a discovery, test a hypothesis, or demonstrate a known fact</p> <p>Impossible - Not able to occur, exist, or be done</p> <p>Interquartile Range – measure of variability; the difference between the first quartile and third quartile of a set of data. It is a way to describe the spread of a set of data or how the data are scattered.</p> <p>Likelihood – how likely, unlikely, equally likely, certain, or impossible an event is to occur</p>

Likely - Such as well might happen or be true; probable

Long Run Relative Frequency - The probability of an event when an experiment is carried out many times

Model - A mathematical representation (number graph, equation, etc) for real world or mathematical objects, properties, actions, or relationships

Outcome – an element in the sample space

Prediction - To state in advance on the basis of observation, experience, or scientific reason

Probability Model - Used to assign probabilities to outcomes of a chance process by examining the nature of the process.

Random Sample – a sample chosen from a population in which each data in the population has an equal chance of being chosen.

Relative Frequency (aka Experimental Probability) -The ratio of the actual number of favorable events to the total possible number of events; often taken as an estimate of probability

Sample Space - The set of all possible outcomes of a random process

Simple Event - An event that contains a single outcome

Simulation - Carrying out a simple experiment to collect data

Survey Data – information (numerical or categorical) collected by asking questions of members of population

Theoretical Probability - The probability that a certain outcome will occur, as determined through reasoning and/or calculation

Tree Diagram - A diagram shaped like a tree used to display sample space by using one branch for each possible outcome

Trial(s) - To conduct tests to assess performance

Uniform Probability Model - A probability model which assigns equal probability to all outcomes

Unlikely - Not likely to happen, be done, or be true; improbable

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Students might be given a spinner or a dice and asked to make observations of favorable outcomes and make choices of a color or number they think will be the most likely result of the spin or roll. Teacher will prompt students with questions regarding likelihood of certain events, at which point students will conduct a “think-pair-share” to discuss their findings.

Number line activity: Students will be given a set of small cards with words and values on them such as impossible, probable, 25%, 3/5, It will snow today, etc. They will draw a number line, labeled from 0 to 1, and glue each card in the appropriate spot. Students should then share and defend results.

Students might be given a brown paper bag with 10 colored cubes in each bag. Each bag will

contain a different distribution of blue, green and yellow cubes. Students will pull a cube from the bag and record the result. The cube will be placed back in the bag and they will repeat this procedure 20 times. Using the results, students will predict the number of each color cube in the bag. Students can switch bags and repeat the procedure.

Students might be put into groups and work through different stations set up around the room (i.e. dice station, coin station, spinner station). Students will perform multiple trials at each station and gather data. Each group will be given a different number of trials to perform (10, 20, 30, etc). They might compare results with other groups or as whole class to discover as the number of trials increases the relative frequency approaches the theoretical probability.

Reference: [http://maths.nayland.school.nz/Year 11/AS1.13 Probability/4 relative freq.htm](http://maths.nayland.school.nz/Year_11/AS1.13_Probability/4_relative_freq.htm)
(Explains definition and gives a simulation of long-run relative frequency with flipping a coin)

Students will be introduced to the game Rock, Paper, Scissors. They will make a tree diagram, using theoretical probability, with all the possible outcomes. Students then play the game 20 times and record the results of the game. A table should be made to show what each player did and the results of the game (win, lose, or tie). Discuss with a partner the differences that may exist between the tree diagram that was developed and the data (in the table) that has been collected from playing. Then, students will brainstorm together why there are discrepancies between these two 'models.' For homework, students will individually write a 'report' on these findings. (Specific requirements can be determined by each teacher)

Students might use two different spinners, one that has numbers and one that has colors. They will have to write out all of the possibilities that may occur from each spinner. Students will then create a tree diagram outlining the various combinations that are plausible. Next, students will be tasked with finding the probabilities of various combinations.

Students might be given the task of picking out their new car. Students can research their favorite car and determine the different options, including color, transmission, engine type, number of doors, etc. Students will then create a model to represent the various combinations and then determine the probability of selecting their desired car at random.

Teacher will have students play a game with a spinner with 8 equal sections. Four sections are grey, three are white, and one is blue. Player A and B use the spinner to play a game. The rules are:

- Player A gets a point if the spinner lands on gray or blue.
- Player B gets a point if the spinner lands on white or blue.
- First player to ten points wins.

Students will play the game and then discuss the results and how/why they were obtained. Students must determine whether or not the game was fair. If it was not, they will then be tasked with making the game “fair” to both players. If/when the students determine that the game was not fair, they will re-create the game- specifying the new design, instructions, etc. so that each player has an equally likely chance of winning.

Supplemental Materials and Resources

Math is Fun – Using and Handling Data - <http://www.mathsisfun.com/data/index.html>

Discovery Education

This website offers a basic lesson that students will learn what probability is; different ways to express probability numerically; and have students learn to solve problems

Shodor

Compare theoretical and experimental probabilities, using dice, cards, spinners, or coin tosses. Three different probabilities can be compared at once.

<http://www.shodor.org/interactivate/activities/CrazyChoicesGame/>

Cereal Toy Investigation

In this activity, one of six different toys is packed in each box of cereal on the assembly line and equal numbers of all six toys are available at all times on the shelves of grocery stores. Students are asked to estimate how many boxes of cereal the average family would have to buy to get all six different toys.

mathwire.com

NRICH enriching mathematics

This page contains a tool for collecting experimental data and a number of problems, which all include interactivities. The interactivities are designed to enable rapid collection of experimental data so the focus in the lesson is on discussion of what the data shows and on finding theoretical probabilities to help support conjectures.

<http://nrich.maths.org/6005>

Literature connection:

Cloudy with a Chance of Meatballs by Judi Barret

The Hunger Games by Suzanne Collins

Interdisciplinary connections:

Science

- Genetics
- Predicting Weather

Social Sciences

- Polls and Samplings
- Population
- Elections
- Immigration

Physical Education

- Sports (odds of making a field goal, odds of winning the coin toss)

Tools/Manipulatives

- Dice, cards, spinners, coins, dart board, colored cubes, marbles
- Cups, paper bags, index cards
- Paper, graph paper, colored pencils, scissors, tape, spinners
- Calculators
- Rulers, compass, protractor

Suggested Formative Assessment Practices/Processes

- Teacher created quizzes
 - *The following are the types of exit tickets you may choose to use in this unit.*

EXIT TICKET: Choose from the following: certain/impossible/likely, to describe the probability.

1. *If you toss a coin it will land either on heads or tails*

2. *If you choose a letter from the word SCHOOL, it will be an 'O'*
3. *If you roll a number cube (1, 2, 3, 4, 5, 6) the number will be greater than 6*
4. *If you roll a number cube (1, 2, 3, 4, 5, 6) the number will be prime*

Differentiation and Accommodations

he randomly chooses one, he thinks that the probability of selecting blue is 1/4. Is he correct?

- Provide graphic organizers

Explain Provide additional examples and opportunities for repetition

EXIT TICKET: Illustrate and explain, using a specific example, how you could use a tree diagram

- Provide tutoring opportunities

to analyze probability. Provide retesting opportunities after remediation (up to teacher and district discretion)

- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments