<table>
<thead>
<tr>
<th>Grade/Subject</th>
<th>Grade 8/ Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grade 7/Accelerated Mathematics</td>
</tr>
<tr>
<td>Unit Title</td>
<td>Unit 1: Real Numbers</td>
</tr>
<tr>
<td>Overview of Unit</td>
<td>Know that there are numbers that are not rational, and approximate them by rational numbers</td>
</tr>
<tr>
<td>Pacing</td>
<td>4-5 days</td>
</tr>
</tbody>
</table>

**Background Information For The Teacher**

- Irrational numbers are a subset of the Real Number System.

![Real Number System Diagram]

- Standard 8.NS.1 and 8.NS.2 in this unit are connected to 8.G.6, 8.G.7, and 8.G.8 in which estimates of irrational numbers are used when developing a conceptual understanding of the Pythagorean Theorem and applying it to find distances on a coordinate plane.

- This is the first time students will be exposed to the radical as mathematical procedure.

- In fourth grade, students will learn how to convert from decimal to fractions and place them on the number line.
In sixth grade, students are introduced to rational numbers including integers.

Standard 8.EE.2 is connected to the Geometry standards in which the understanding of square roots is embedded in the development of the Pythagorean Theorem, and representations of cube numbers are embedded within applications of volume in a later unit.

### Essential Questions (and Corresponding Big Ideas)

**Why do I need to understand the types of numbers found in the real number system?**

- *Real world situations require the use of various forms of real numbers, including scientific notation.*

**How do I determine the best numerical representation for a given situation?**

- *All real numbers can be compared, classified, and expressed in various forms.*

### Core Content Standards

<table>
<thead>
<tr>
<th>Core Content Standards</th>
<th>Explanations and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Students expand their knowledge of the Real Number System to include irrational numbers. An irrational number is a decimal whose expansion does not terminate or repeat. Irrational numbers cannot be written in</td>
<td>8.NS.1 Students can use graphic organizers to show the relationship between the subsets of the real number system.</td>
</tr>
</tbody>
</table>

8.NS.1

**Real Numbers**

All real numbers are either rational or irrational.
Mathematics/Grade 8  Unit 1: Real Numbers

fraction form. Using decimal expressions, students compare rational numbers and irrational numbers to show that rational number expansions repeat and irrational number expansions do not. The notation “…” means “continues indefinitely without repeating.” For example, 0.\overline{3} is a rational number that repeats but \pi = 3.1415\ldots does not repeat.

1. Let \(x = 0.555\)
2. Multiply both sides so that the repeating digits will be in front of the decimal. In this case, one digit repeats so both sides are multiplied by 10, giving 10x = 5.555\ldots
3. Subtract the original equation form the new equation.
   \[10x = 5.555\ldots - x = 0.555\ldots \]
   \[9x = 5\]
4. Solve the equation by dividing both sided of the equation by 9.
5. \(x = \frac{5}{9}\)

What the teacher does:
- Pose questions such as the following: “Will a rational number eventually repeat?” Can you find a rational number that does not repeat? Use this discussion as an introduction for students to discover irrational numbers.
- Have students reason about the inclusive nature of the subsets of Real Number System and complete a Venn diagram of the Real Number System.
- Access prior knowledge about converting fractions to decimals. Relate the concept to changing the decimal expansion of a repeating decimal expansion of a repeating decimal into a fraction and a fraction into a repeating decimal. Provide examples.

What the students do:
- Clarify understanding of rational numbers as repeating or terminating through discussion about irrational numbers.
- Recognize and use the notation for decimal expansions of irrational numbers.
- Complete a Venn diagram to clarify their understanding of the Real Number System as the set of numbers made up of the rational and the irrational numbers.
- Convert decimal expansions into equivalent fractions using an algorithm.
- Use strategies other than conversions for some decimal expansions; for example, after exploring the ninths, students may remember the repeating pattern
  \[\left(\frac{1}{9} = 0.\overline{1}, \frac{2}{9} = 0.\overline{2}\right)\]
- Recall common fractions such as \(\frac{3}{4} = 0.75\).

Misconceptions and Common Errors:
Some students have difficulty understanding the relationship of the subsets of the Real Number System with a Venn diagram. Try hands-on approach using boxes or bags that fit inside one another to represent the subsets.

Some students need more practice than others converting repeating decimals to equivalent fractions. This can be done over time with mini-practice sessions weekly.

8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \(\pi^2\)). For example, by truncating the decimal expansion of \(\sqrt{2}\), show that \(\sqrt{2}\) is
between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Students compare irrational numbers and locate them on a number line by finding their rational approximations. Find rational approximations by creating lists of numbers by answering the following question: Between which two numbers will you find \( \sqrt{2} \)? Since \( 1^2 = 4 \), it is between 1 and 2. To be more precise, is it closer to 1 or 2? Systematically square 1.1, 1.2, 1.3, 1.4... 1.9. Between which two numbers do you find 2? Repeat the process until you have the degree of precision you are seeking.

<table>
<thead>
<tr>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.21</td>
<td>1.44</td>
<td>1.69</td>
<td>1.96</td>
<td>2.25</td>
<td>2.56</td>
<td>2.89</td>
<td>3.24</td>
<td>3.61</td>
<td>4</td>
</tr>
</tbody>
</table>

8.NS.2 Students can approximate square roots by iterative processes.

Examples:

- Approximate the value of \( \sqrt{5} \) to the nearest hundredth.
  
  Solution: Students start with a rough estimate based upon perfect squares. \( \sqrt{5} \) falls between 2 and 3 because 5 falls between \( 2^2 = 4 \) and \( 3^2 = 9 \). The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. \( \sqrt{5} \) falls between 2.2 and 2.3 because 5 falls between \( 2.2^2 = 4.84 \) and \( 2.3^2 = 5.29 \). The value is closer to 2.2. Further iteration shows that the value of \( \sqrt{5} \) is between 2.23 and 2.24 since 2.23² is 4.9729 and 2.24² is 5.0176.

- Compare \( \sqrt{2} \) and \( \sqrt{3} \) by estimating their values, plotting them on a number line, and making comparative statements.

  Solution: Statements for the comparison could include:

  - \( \sqrt{2} \) is approximately 0.3 less than \( \sqrt{3} \)
  - \( \sqrt{2} \) is between the whole numbers 1 and 2
  - \( \sqrt{3} \) is between the whole numbers 1 and 2

  What the students do:

  - Reason abstractly to determine where to place an irrational number on the number line. Students begin to focus on the precision required of the task. It is not unreasonable to expect them to ask how precise they should be for the...
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Students learn that squaring and cubing numbers are the inverse operations to finding square and cube roots. This standard works with perfect squares and perfect cubes, and students will begin to recognize these numbers. Equations should include rational numbers such as $x^2 = \frac{1}{4}$ and $x^3 = \frac{1}{64}$ and fractions were both the numerator and denominator are perfect squares or cubes.

\[
\begin{align*}
x^2 &= \frac{1}{4} \\
\sqrt{x^2} &= \pm \frac{\sqrt{1}}{\sqrt{4}} \\
x &= \pm \frac{1}{2}
\end{align*}
\]

Square roots can be positive or negative because $2 \times 2 = 4$ and $-2 \times -2 = 4$.

**What the teacher does:**
- Introduce squaring a number and taking the square root as inverse operations, providing students opportunities to practice squaring and taking roots.
- Repeat the previous instruction for cubes and cube roots, also including fractions where the numerator and denominator are both perfect cubes.
- Relate perfect numbers and perfect cubes of geometric square and cubes using square tiles and square cubes to build given exercise. Tenths? Hundredths?
- Look for and express regularity in the repeated reasoning used in finding approximations of irrational numbers.
- Reason abstractly as they become more familiar with the process to find approximations of irrational numbers to streamline the algorithm.
- Express thinking in writing to clarify understanding about how to find precise approximations of irrational numbers.

**Misconceptions and Common Errors:**
When rational numbers written in decimal form have more than three digits that repeat, some students stop the division process and call it an irrational number. These students need to be encouraged to preserve with the division until they are convinced there is no repeat. These students may not have a clear understanding of rational numbers as numbers that can be written in fraction form. This fact should be made explicit during instruction. To help students who become overwhelmed with the process to approximate irrational numbers, suggest an organized format. For example, set up three columns with questions that need to be answered for each. Some students may need the template at first.

- _____ falls between which two whole numbers?
- Is _____ closer to _____ or _____?
- Is _____ closer to _____ or _____?

**8.EE.2 Examples:**

- $3^2 = 9$ and $\sqrt{9} = \pm 3$
- $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{3\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$
- Solve $x^2 = 9$
  
  **Solution:** $x^2 = 9$
  \[
  \begin{align*}
  \sqrt{x^2} &= \pm \sqrt{9} \\
x &= \pm 3
  \end{align*}
  \]
the numbers. A square root is the length of the side of a square, and a cube root is the length of the side of a cube.

- Encourage students to find patterns within the list of square numbers and then with cube numbers.
- Facilitate a class discussion around the question, “In the equation \( x^2 = p \), when can \( p \) be a negative number?” Students should come to the conclusion that it is not possible.
- Discuss non-perfect squares and non-perfect cubes as irrational numbers such as \( \sqrt{2} \).

\[
\begin{align*}
\text{Solve } & x^3 = 8 \\
\text{Solution: } & x^3 = 8 \\
& 3\sqrt{x^3} = 3\sqrt{8} \\
& x = 2
\end{align*}
\]

What the students do:
- Recognize perfect squares and perfect cubes.
- Solve equations containing cube and square roots.
- Discover and explain the relationship between square and cube roots and the sides of a square and the edges of a cube, respectively, by using hands-on materials.
- Reason that non-perfect squares and non-perfect cubes are irrational, including the square root of 2.

Misconceptions and Common Errors:
It is important for students to have multiple opportunities and exposures with perfect cubes. This is a new concept in the curriculum and many students struggle with finding cube roots. A common misconception for cube roots is that any number times 3 is a perfect cube. Building larger cubes from smaller ones gives students a visual that they can rely on.

Standards for Mathematical Practice

In this cluster students expand their knowledge of the real numbers to include irrationals. Students reason with approximations of irrational numbers and explain how to get more precise approximations. Students compare and locate irrational numbers on a number line. In grade 7 students convert fractions into their decimal expansions. Eighth graders convert the decimal expansions into fraction form. 8.NS.1, 8.NS.2

MP2. Reason abstractly and quantitatively.

Explanations and Examples

Students are reasoning as they explain how to get more precise approximations of irrational numbers.
K-U-D

**KNOW**
*Facts, formulas, information, vocabulary*

- There are numbers that are not rational called “irrational”.
- Irrational numbers are a subset of the Real Number System.

**DO**
*Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*

- KNOW rational and irrational numbers
- UNDERSTAND decimal expansion
- SHOW decimal expansion repeats
- CONVERT repeating decimal expansion to a rational number
- USE
  - rational approximations of irrational numbers
    - COMPARE sizes of rational numbers
    - LOCATE rational numbers approximately on a number line
    - ESTIMATE value of expressions
  - square root and cube root symbols
- EVALUATE
  - square roots of perfect squares
  - cube roots of perfect cubes
Every number has a decimal representation:
- Irrational decimals are non-repeating and non-terminating
- Rational number decimals eventually terminate or repeat.
- Irrational numbers can be approximated for comparing and ordering them.
- A perfect square is a number in which the square root is an integer.
- A perfect cube is a number in which the cube root is an integer.
- √2 is irrational.
- Equivalent forms of an expression allow for efficient problem solving.
- Estimation can be used as a means for predicting & assessing the reasonableness of a solution.

**UNDERSTAND**

*Big ideas, generalizations, principles, concepts, ideas that transfer across situations*

In the real number system, numbers can be defined by their decimal representations.

**Common Student Misconceptions for this Unit**
- Students may not continue to divide to recognize a repeating decimal.
- Students may think that squaring means to multiply by 2.
- Students may think that finding the square root means to divide by 2.
• Students may think that cubing means to multiply by 3.
• Students may think that finding the cube root means to divide by 3.
• Students do not consider the negative square roots of the number. For example, $\sqrt{9} = 3$ instead of $\sqrt{9} = \pm 3$
• On a number line, students may struggle with plotting negative values. For example, $-1.5$ would be placed between 0 and $-1$.
• On a number line, students will struggle with comparing negative numbers to negative numbers. For example $-6 > -5$.
• Students may divide the denominator by the numerator (instead of numerator by denominator) when converting a fraction to a decimal, resulting in a number greater than one.

<table>
<thead>
<tr>
<th>Unit Assessment/Performance Task</th>
<th>DOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1 Test</td>
<td></td>
</tr>
<tr>
<td>Unit 1 Performance Task “Rational or Irrational Reasoning”</td>
<td></td>
</tr>
<tr>
<td>Unit 1 Performance Task “Let’s Approximate Roots”</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary**

- Real numbers
- Rational numbers
  - Natural numbers
  - Whole numbers
  - Whole numbers
  - Integers
- Irrational numbers
- Repeating decimals
- Terminating decimals
- Decimal expansion
- Square root
- Perfect square
- Cube root
- Perfect cube
- Approximate

### Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

**Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)**

Use Station 1 activity to group numbers into terminating, repeating, or "other" decimals, discuss different types of numbers, especially square roots; HW: find 3 square roots that are terminating and 3 that are not

*In cooperative groups, students will develop the definition of rational and irrational numbers by investigating sets of numbers. (integers, naturals, whole, rational, irrational)*

- Understand and show the relationship between the subsets of the real number system (rational and the subsets of rational numbers and irrational numbers and irrational numbers)
- Identify and know which numbers belong to which subsets of the real number system
- Show decimal expansion of a rational number repeats or terminate
- Convert decimal expansions (both repeating and terminating) to a rational number

Introduce real number system using graphic organizer, then have students take a given number and put it on giant number system; "exit ticket" for homework

*Teachers will provide students with the area of various size squares and students will find side...*
lengths.
- Understand and know perfect squares and square roots (up to 144, and including 225, 400, and 625) and that they are part of the real number system
- Estimate the rational value of square roots (using mental math)
- Using square root symbols, represent and evaluate solutions to equations

Students will use knowledge of square roots to devise a definition of cube roots. With this definition, students will identify 2 cube roots.
- Understand and know perfect cubes and cube roots (including 1, 8, 27, 64, 125, 1000) and that they are part of the real number system
- Using cube roots symbols, represent and evaluate solutions to equations

Compare and locate various types of Rational and Irrational numbers on the number line (exact and estimated)
Using real world examples (baseball statistics, Olympic results, etc.) students will compare, order, and locate on the number line.
- Compare sizes of rational numbers
- Use rational approximations of irrational numbers such as the $\sqrt{2}$ and $\sqrt{3}$, locate on the number line, and make comparative statements
- Locate positive and negative rational and irrational numbers exactly and approximately on the number line.

Supplemental Materials and Resources

Literature connection:
- The Square Root of 2 By: David Flannery
- Square Root By: Derek Beaulieu
- Short task on the number system:
http://map.mathshell.org/materials/tasks.php?taskid=398#task398

- Pop up game - Rational or Irrational Number?: Http://www.quia.com/pop/37541.html?AP_rand=1107411821
- When not knowing math can cost you $15,000: http://www.youtube.com/watch?v=BbX44YsQ2I
  This is a clip from a television game show where the contestant answers a question about square numbers incorrectly and loses $15000.

<table>
<thead>
<tr>
<th>Tools/Manipulatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
</tr>
<tr>
<td>Number line</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Suggested Formative Assessment Practices/Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher created exit slips, teacher created quizzes</td>
</tr>
</tbody>
</table>

Revised March 2017
<table>
<thead>
<tr>
<th>Instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Provide graphic organizers</td>
</tr>
<tr>
<td>• Provide additional examples and opportunities for repetition</td>
</tr>
<tr>
<td>• Provide tutoring opportunities</td>
</tr>
<tr>
<td>• Provide retesting opportunities after remediation (up to teacher and district discretion)</td>
</tr>
<tr>
<td>• Teach for mastery not test</td>
</tr>
<tr>
<td>• Teaching concepts in different modalities</td>
</tr>
<tr>
<td>• Adjust homework assignments</td>
</tr>
</tbody>
</table>