

Grade/Subject	Grade 8/ Mathematics Grade 7/ Accelerated Mathematics
Unit Title	Unit 2: Exponential Expressions and Equations
Overview of Unit	Work with radicals, integers exponents, and scientific notation
Pacing	32 days

Background Information For The Teacher
<ul style="list-style-type: none"> • In fifth grade, students will learn how to multiply and divide by powers of ten with decimals. • In sixth grade, students are introduced to the concept of exponents through formulas but do not use the word exponents. • In eighth grade, students will be introduced to the terms exponents and scientific notation. • Build understanding of the properties of integer exponents through analyzing number patterns and relationships. Students should discover the integer exponent properties without being given the properties. • Standard 8.EE.2 is connected to the Geometry standards in which the understanding of square roots is embedded in the development of the Pythagorean Theorem, and representations of cube numbers are embedded within applications of volume in a later unit. • Use powers of 10 and knowledge of the base 10 number system to decompose and recompose numbers in standard and scientific notation for estimating and approximating mental computations when dealing with values in various terms.

Essential Questions (and Corresponding Big Ideas)

Why do I need to understand the types of numbers found in the real number system?

- *Real world situations require the use of various forms of real numbers, including scientific notation.*

Core Content Standards	Explanations and Examples																		
<p>8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i></p> <p>Students learn how to compute using integer exponents building on their earlier experiences with adding and subtracting integers. For any non-zero real numbers a and b and integers n and m, the properties of integer exponents are as follows:</p> <p>Properties of Exponents</p> <table border="1"> <thead> <tr> <th>RULE</th> <th>PROOF</th> </tr> </thead> <tbody> <tr> <td>1. $a^m a^n = a^{m+n}$</td> <td>$2^2 \cdot 2^3 = 2^{2+3} = 32$</td> </tr> <tr> <td>2. $(a^m)^n = a^{mn}$</td> <td>$(2^3)^2 = 2^{3 \cdot 2} = 64$</td> </tr> <tr> <td>3. $(ab)^n = a^n b^n$</td> <td>$(2 \cdot 3)^2 = (2^2)(3^2) = 36$</td> </tr> <tr> <td>4. $a^{-n} = 1/a^n$</td> <td>$2^{-2} = 1/2^2 = 1/4$</td> </tr> <tr> <td>5. $(a/b)^m = a^m/b^m$</td> <td>$(1/2)^2 = 1^2/2^2 = 1/4$</td> </tr> <tr> <td>6. $(a/b)^{-n} = (b/a)^n$</td> <td>$(2/4)^{-2} = (4/2)^2 = 4$</td> </tr> <tr> <td>7. $a^0 = 1$*</td> <td>Any number⁰ = 1</td> </tr> <tr> <td>8. $a^m/a^n = a^{m-n}$</td> <td>$2^5/2^2 = 2^{5-2} = 2^3 = 8$</td> </tr> </tbody> </table> <p>* unless a = 0</p> <p><u>What the teacher does:</u></p> <ul style="list-style-type: none"> • Introduce the laws of integer exponents one at time. Use a conceptual approach as opposed to asking students to memorize the rules. • Provided examples of the processes that lead to the rules for each law, such as $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$. Allow students to try a few similar expressions to see if they can find the solution and post a rule or property they may discover. • Provide examples: $6^2 \cdot 7^2 = (6 \times 6)(7 \times 7) = (6 \times 7)(6 \times 7) = (6 \times 7)^2$. Ask students to try several of their own to see if they can discover a rule of property. 	RULE	PROOF	1. $a^m a^n = a^{m+n}$	$2^2 \cdot 2^3 = 2^{2+3} = 32$	2. $(a^m)^n = a^{mn}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$	3. $(ab)^n = a^n b^n$	$(2 \cdot 3)^2 = (2^2)(3^2) = 36$	4. $a^{-n} = 1/a^n$	$2^{-2} = 1/2^2 = 1/4$	5. $(a/b)^m = a^m/b^m$	$(1/2)^2 = 1^2/2^2 = 1/4$	6. $(a/b)^{-n} = (b/a)^n$	$(2/4)^{-2} = (4/2)^2 = 4$	7. $a^0 = 1$ *	Any number ⁰ = 1	8. $a^m/a^n = a^{m-n}$	$2^5/2^2 = 2^{5-2} = 2^3 = 8$	<p>8.EE.1 Examples:</p> <ul style="list-style-type: none"> • $\frac{4^3}{5^2} = \frac{64}{25}$ • $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ • $\frac{4^{-3}}{5^2} = 4^{-3} \cdot \frac{1}{5^2} = \frac{1}{4^3} \cdot \frac{1}{5^2} = \frac{1}{64} \cdot \frac{1}{25} = \frac{1}{16,000}$ <p><u>What the students do:</u></p> <ul style="list-style-type: none"> • Discover the properties of integer exponents by making sense of the examples presented. For example, $(3x7)^4 = (3x7)x(3x7)x(3x7)x(3x7) = (3x3x3x3)x(7x7x7x7) = 3^4x7^4$ with the rule discovered as $(ab)^n = a^n b^n$. • Generate equivalent expressions in simplest form for products and quotients of numbers with integer exponents having the same bases. <p><u>Misconceptions and Common Errors:</u></p> <p>Students often confuse the rules. This occurs primarily when students are taught to memorize the rules rather than understand what is happening in the properties by working with numerical expressions as in the suggestions above. It is important to present examples and let students discover what the rules are. Then students should be encouraged to write their reasoning so they can clarify the explanations for themselves.</p>
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1. $a^m a^n = a^{m+n}$	$2^2 \cdot 2^3 = 2^{2+3} = 32$																		
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8. $a^m/a^n = a^{m-n}$	$2^5/2^2 = 2^{5-2} = 2^3 = 8$																		

- Have students practice the properties by generating equivalent expressions and writing them in simplest form such as $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
- Provide examples to lead students to discover how a negative exponent translates to a positive exponent in the denominator of a fraction, such as how $3^{-3} = \frac{1}{3^3}$.
- Assign a project for students to design and create posters summarizing the rules they discover to hang in the classroom of the properties of exponents with integers.

8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

This standard emphasizes scientific notation. Students write very large and very small numbers in scientific notation using positive and negative exponents. For example, 123,000 written in scientific notation is 1.23×10^5 , and 0.0008 written in scientific notation is 8×10^{-4} . When mastered, students use the skill to determine how many times larger (or smaller) one number written in scientific notation is than another. To compare, if the exponents increases by 1, the value increases 10 times. In the example of the U.S. and world populations, the exponent is increased by 1, and the 7 is a little more than 2 times 3. So 2×10 makes for 20 times larger.

What the teacher does:

- Introduce examples of very large and very small numbers in contexts. Contexts can be found in sources such as government statistics websites, population sizes, land mass in area, and science. Ask students why writing very large and very small numbers in scientific notation would be beneficial. Who would use it?
- Provide students with the opportunity to research very large and very small numbers and present them written in scientific notation, along with the contexts, to the class. Discuss why these numbers are considered estimates. Keep a bank of these numbers and their contexts for students to use at a later time to create real-world problems.
- Provide contextual problems for students to compare numbers written in scientific notation.
- Use the bank of numbers created by the students to ask

8.EE.3

Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.

What the students do:

- Understand the benefits of using scientific notation.
- Research to find examples of very large and very small numbers.
- Write very large and very small numbers in scientific notation.
- Understand that some numbers written in scientific notation are estimates. Explain why that is true.
- Compare numbers written in scientific notation to determine how many times larger (or smaller) one number written in scientific notation is than another.

about how some of the numbers are related to one another by comparing similar contexts.

8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

This standard builds on previous standards as now students use what they know about scientific notation and properties of integer exponents to solve problems. Quantities in the problems can be expressed in scientific notation and decimal form. Students focus on the size of the measurement to determine which units are appropriate for the context such as millimeters for very small quantities. This standard also class for students to use technology and be able to interpret the scientific notation used. The teacher needs to check the class calculators to be familiar with the notation used by those particular calculators, as the notation used by calculators to express scientific notation is not standard.

What the teacher does:

- **Pose** problems that require students to perform operations with numbers written in scientific notation.
- Present problem-solving opportunities for students to choose correct units of measurement when working with very large or very small numbers including making conversions between units such as in the following problems: "An average ant is 10^{-1} centimeters long. If you laid ants end to end, how many would it take to make a line from New York City to Disney World? The distance to Disney World from NYC is 1,513 kilometers.
- Provides students with calculators. Give them a calculation to perform that results in a number displayed in scientific

Misconceptions and Common Errors:

Students often confuse a very large number for a small number when written in scientific notation such as 4,000,000 for 4×10^{-6} . This usually is a result of students trying to memorize a rule about moving a decimal point to the left or the right. Instead of teaching a rule, rely on students' background knowledge of negative exponents. Before rewriting a number in standard form, look to the exponent to determine whether it is a small or large number. This can also be used as a check.

Students who do not understand the properties of exponents also make errors in computation with scientific notation. Teacher may need to review these properties.

8.EE.4

Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of $2.45E+23$ is 2.45×10^{23} and $3.5E-4$ is 3.5×10^{-4} . Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

Mathematics/Grade 8 Unit 2: Exponential Expressions and Equations

<p>notation. Facilitate a large group discussion about what the notation in the display means and how it is a form of scientific notation. Provide other opportunities for students to interpret scientific notation expressed with technology. Ask, “Do all calculators use the same display for scientific notation?”</p>	<p><u>What the students do:</u></p> <ul style="list-style-type: none"> • Perform operations with numbers written in scientific notation. Solve both mathematical and real-world problems. • Choose correct units for very large and very small numbers when solving problems. • Discover and interpret the rules for scientific notation displayed on a given calculator. <p><u>Misconceptions and Common Errors:</u></p> <p>When performing operations with numbers in scientific notation, such as $(7 \times 10^5) \times (18 \times 10^9)$, some students will be overwhelmed with keeping track of what they should do. Encourage these students to color code the numbers such as highlighting the number in exponential form in the given example so students remember to work them together.</p>
<p>Standards for Mathematical Practice</p>	<p>Explanations and Examples</p>
<p>Work with radical and integer exponents. 8.EE.1, 8.EE.3, 8.EE.4 In this cluster students learn how to compute with integer exponents. Students build on what they have learned about square roots to solve equations in the form of $x^2 = p$ and $x^3 = p$, where p is a positive rational number, evaluating perfect square and perfect cube roots. Students learn how to express very large and very small numbers in scientific notation and express how many times larger or smaller one number written in scientific notation is than another. Students use the properties of integer exponents to perform operations with numbers written in scientific notation. Students interpret numbers written in scientific notation using technology.</p> <p>MP2. Reason abstractly and quantitatively.</p>	

Mathematics/Grade 8 Unit 2: Exponential Expressions and Equations

<p>MP5. Use appropriate tools strategically.</p> <p>MP6. Attend to precision.</p> <p>MP7. Look and make use of structure.</p>	<p>Students use reasoning to express how many times larger (or smaller) one number is than another when both are expressed in scientific notation.</p> <p>Students learn to read scientific notation as expressed by technology.</p> <p>Students compute with integer exponents and numbers in scientific notation accurately.</p> <p>Students will see and make use of patterns in scientific notation in square and cube numbers.</p>
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K-U-D	
<p>KNOW <i>Facts, formulas, information, vocabulary</i></p>	<p>DO <i>Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)</i></p>
<p>Properties of integer exponents.</p> <p>The base ten number system can be applied to represent very large and very small numbers using powers of 10.</p> <p>Equivalent forms of an expression allows for efficient problem solving.</p> <p>Strategies for computing with numbers expressed in scientific notation.</p> <p>Estimation as a means for predicting & assessing the reasonableness of a solution.</p>	<ul style="list-style-type: none"> • USE integer power of 10 • ESTIMATE large or small quantities • EXPRESS magnitude of numbers using powers of 10 • REPRESENT solutions to equations • Use scientific notation to REPRESENT very large and very small numbers • CHOOSE units of appropriate size • KNOW and APPLY properties of integer exponents • GENERATE equivalent numerical expressions • CALCULATE/CONVERT numbers expressed in scientific notation/decimal form • INTERPRET scientific notation generated by technology • Solve problems that use equations involving variables that are squared or cubed.
<p>UNDERSTAND <i>Big ideas, generalizations, principles, concepts, ideas that transfer across situations</i></p>	

The properties of number systems and their relationships remain consistent when applied to integer exponents.

Common Student Misconceptions for this Unit

- Students may think that $x^0 = 0$ instead of 1.
- Students may think that $4^{-2} = -8$ or that a negative exponent will always result in a negative number.
- Common mistake may be: $-2x^{-2} = \frac{1}{2x^2}$
- Students may interchange the negative and positive exponents in scientific notation.
- Students may not recognize that the first number does not consist of a single digit to the left of the decimal.
- Students may move the decimal in the wrong direction when converting from scientific notation to standard notation and vice versa.

Unit Assessment/Performance Task	DOK
Unit 2 Test Unit 2 Performance Task “Giant Burger” Unit 2 Performance Task “Sierpinski Triangles” Unit 2 Performance Task “Ant & Elephant”	

Vocabulary

- Base
- Bivariate Data
- Distributive Property
- Equation
- Equivalent

- Evaluate
- Exponent
- Exponential notation
- Expression
- Linear Equation
- Numerical Expression
- Power
- Scientific Notation
- Simultaneous Equations
- Variable

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Section 1.1

Day 1: Notes and class discussion on understanding exponential notation - base, exponent, write in exponent notation, expand and evaluate

Day 2: Writing prime factorization and compound interest formula

Section 1.2

D1: Notes and discussion on Product of Powers

D2: Practice with more complicated expressions

D3: Notes and discussion on Quotient of Powers

D4: Practice with more complicated expressions and put together

D5: Word problems from section

Section 1.3

D1: Notes and discussion on Power of a Power property

D2: Simplify more complicated expressions

Quiz

Section 1.4

D1: Classwork worksheet on Power of Product and then notes/discussion

D2: Finish classwork worksheet on Power of a Quotient then notes/discussion - put together

D3: WS 1 to review skill

Section 1.5

D1: Classwork WS on Zero Exponents and then notes/discussion

D2: Classwork WS on Negative Exponents and then notes/discussion

D3: Put together more complicated expressions

Section 1.6

D1: Notes/discussion on square and cube roots, equations

D2: Word problems with square and cube roots

Chapter 1 Assessment

Section 2.1

D1: Notes and discussion on Scientific Notation - convert both ways

D2: Notes and discussion on comparing Scientific Notation Numbers

Section 2.2

D1: Notes and discussion on Add/Subtract with same exponent

D2: Notes and discussion on Add/Subtract with different exponents

Introduce the prefix system.

Quiz – Converting and Adding/Subtracting

Section 2.3

Notes and discussion on multiply/divide

Chapter 2 Assessment

Supplemental Materials and Resources

- Real-world data to convert to scientific notation (ex. solar system)
- “A Million Dollars” Using properties of integers to solve problems:
<http://map.mathshell.org/materials/tasks.php?taskid=360#task360>
- Adding signed numbers- matching
game:<http://learningwave.com/chapters/integers/numline.html>
- Lesson on Estimating lengths using scientific notation:
<http://map.mathshell.org/materials/lessons.php?taskid=414#task414>
- “100 people in the World” Task: estimate using scientific notation
<http://map.mathshell.org/materials/tasks.php?taskid=359#task359>
- Scientific Notation online quiz: <http://www.quia.com/quiz/113578.html>
- Practice with Scientific Notation: http://proton.csudh.edu/lecture_help/scinot.html

Literature connection:

- Look through newspapers/magazines and have students find articles where scientific notation can be useful
- Scientific Notation in Everyday Life
<http://www.math.toronto.edu/mathnet/questionCorner/scinot.html>

Interdisciplinary Connections:

- Science:
 - Using scientific notation in distances in space, molecules, chemistry,
 - Using exponents in acceleration, velocity
 - Newton’s Laws of motion
 - Biology- exponential growth of bacteria

- Social Studies:
 - Finding population of the geographic region being studied in terms of scientific notation
 - Finding the area of the geographic region being studied in terms of scientific notation
 - Analyzing census

Tools/Manipulatives

Differentiation and Accommodations

Suggested Formative Assessment Practices/Processes

Teacher created exit slips, teacher created quizzes

Calculator

- Provide graphic organizers
- Provide additional examples and opportunities for repetition
- Provide tutoring opportunities
- Provide retesting opportunities after remediation (up to teacher and district discretion)
- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments