Grade/Subject	Grade 8/ Mathematics
Unit Title	Unit 3: Linear Relationships 3A: Solving linear equations, understanding rate of change. 3B: Systems of Equations
Overview of Unit	In this unit, students will understand the connections between proportional relationships, lines, and linear equations. They will also analyze and solve linear equations with distributing, combining like terms, and variables on both sides. With functions, students will define, evaluate, and compare functions and use functions to model relationships between quantities. Students will demonstrate their understanding through tables and graphs. Included are examples from real world applications and verbal descriptions. Previously, these topics were included in the Algebra I curriculum.
Pacing	38 days

Background Information For The Teacher
Students have been graphing points on the coordinate plane to solve real world problems since the fifth grade. In the sixth grade, they reasoned about and solved one variable equations and inequalities. This included the use of the distributive property. In the seventh grade, students learned to represent proportional relationships by equations. They also explained what a point on the graph of a proportional relationship means in terms of the situation (ex. Unit rate). Students worked with the concept of slope without identifying it as slope. In the seventh grade, students also continued to solve equations including one-step, two-step, and multi-step including distributive property. They do not solve equations with variables on both sides of the equal sign. In this unit, students will represent proportional linear relationships in tables, graphs and scenarios. They will interpret slope in terms of the context of the real world application or mathematical situation. For the first time students will be using slope intercept form to graph linear equations on the coordinate plane. They will be exploring patterns, generalizing patterns and developing informal notions of a variable as a quantity that changes will lead to the informal study of functions as special relationships between

variables. Within this exploration students will encounter nonlinear and linear functions. Students will also analyze and solve pairs of simultaneous linear equations. They will understand what the solution to a system of two linear equations means use systems to solve mathematical and real world problems. Emphasis is put on the graphical method and the elimination method (using manipulatives). Students are introduced to substitution. Students will encounter solving systems of equations (by all three methods) again in Algebra I but at a much deeper and abstract level.

Essential Questions (and Corresponding Big Ideas)

How is mathematics used to measure, model and calculate change?

There are many real world applications for rate of change and solving linear relationships. Linear relationships can be modeled by algebraic, tabular, graphical, mathematical or verbal descriptions.

How are the characteristics of a linear relationship relevant in a real world situation?

Rate of change (slope) and the y-intercept of a linear relationship define important applications in real world situations.

What types of solutions can linear equations have?

A linear equation can have one solution (consistent), no solution (inconsistent), or infinitely many solutions.

How do you solve a system of equations and what does the solution mean?

You can solve systems of equations by using the elimination method, substitution method, and the graphical method. A unique solution of a system represents the point of intersection of the two lines, a system with an inconsistent solution (no solution) means that the lines are parallel, and a dependent system (infinitely many solutions) means that the lines have the same slope and same y-intercept.

Core Content Standards	**Explanations and Examples**
8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a*	8.EE.5. Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs. Example:

distance-time equation to determine which of two moving objects has greater speed.

Students build on their work from Grade 6 with unit rates and their work with proportional relationships in Grade 7 to compare graphs, tables, and equations of linear (proportional) relationships. Students identify the unit rate as slope in graphs, tables and equations to compare proportional relationships presented using different representations. For example, compare the unit rate in a problem about a phone bill presented in graphic form on a Cartesian plane to a phone bill from a different company where the unit rate can be found represented in an equation or table.

What the **teacher** does:

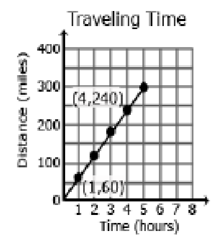
- Present a single, graphed proportional relationship to the class. Facilitate a class discussion about the unit rate, using students' background knowledge, and interpret the unit rate as the slope of the line.
- Presents a second, related, proportional relationship, written in a different form such a table or an equation. Facilitate a class discussion about how to compare the two situations. Use questions such as, What is happening in each situation? How are they the same? Different? How can we tell? What about the slopes? What do they tell us? What can we do to help us compare the slopes (graph the second relationship)?
- Provide opportunities for students to compare proportional relationships and write their conclusion using precise mathematical vocabulary.

8.EE.6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Students gain additional knowledge about slope in this standard as they use similar triangles to explain how the slope m of a line is the same between any two points on a given non-vertical line. Students understand positive/negative slopes, 0 slope, and undefined slopes.

• Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1:



Scenario 2:

$y = 50x$
x is time in hours
y is distance in miles

What the **students** do:

- Make use of structure of a representation to compare different representations. For example, students will find that if a problem asks them to compare the unit rates for information presented in a table and information presented in equation form, graphing both of them will make the comparison easier.
- Compare proportional relationships presented in different forms (graphs, tables, equations, verbal descriptions) and explain comparisons in writing using clear and precise mathematical language. Comparisons will include slope interpreted in context of the relationships.

Misconceptions and Common Errors:

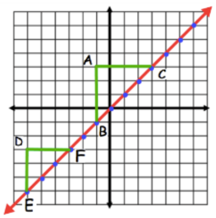
Errors occur when students are overwhelmed by being presented with too much information at a time. Encourage students having difficulty making the comparisons to work with one relationship at a time. Graphing may be a difficult skill for some students. Using graph paper larger than 1 cm for these students so they can see the unit rate easier.
Students who are overwhelmed can also be helped by using graphs of experiences that are familiar to them. This makes the information more accessible so students can better understand and interpret proportional relationships.

8.EE.6. Example:

Through the use of similar triangles, teachers lead students to derive the general equation (y=mx + b) of a line and discover that m is the slope and b is the y-intercept.

What the **teacher** does:

- Facilitate a class discussion about two similar triangles as in the example below:

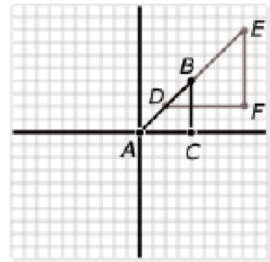


- Explain slope as rise over run (rise is the vertical distance and run is the horizontal distance) by having students see that $\overline{ED}$ rise is 3 units and $\overline{DF}$ run is 3 units for a ratio of 3 to 3 or written as $\frac{3}{3}$. On a similar triangle BCA, $\overline{BA}$ is 4 units and $\overline{AC}$ is 4 units for a ratio 4 to 4, which is the same as 3 to 3. Have students create other pairs of similar triangles to convince themselves that the slope of a line is the same between any two points on a non-vertical line.
- Challenge students to find the slope of a horizontal line (0) and vertical line (undefined) and explain their reasoning.
- Demonstrate how to find a slope using the formula $\frac{y_1 - y_2}{x_1 - x_2}$.
- Lead students to discover the equation y=mx for a line that goes through the origin and y = mx + b for a line that goes through point b. Note that when b = 0, y=mx. The y-intercept is b.

8.EE.7. Solve linear equations in one variable.

a. Give examples of linear equations in

- Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation.



What the **students** do:

- Explain, orally and/or in writing, using similar triangles, why the slope of a line is the same between any two points on a non-vertical line. Use clear and precise language.
- Discover that b is the y-intercept and m is the slope in the general equation for a line, y = mx + b.
- Determine the slope of a line from a graph, table, or linear equation.
- Explain orally and /or in writing how proportional realtionships, lines, and linear equations are related.

Misconceptions and Common Errors:

A common error students make is to misuse the formula for finding the slope of a line given two points. They use x—y or use the difference of the x coordinates divided by the difference in the y coordinates. Look for these common errors. Focus students' attention on the errors by using error analysis tasks. For example Jed used the following equation to find the slope of a line: $\frac{x_1}{y_1} - \frac{x_2}{y_2}$. Find Jed's mistake and correct it.

one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. This standard has students solving linear equations. It is explained by 8.EE.7 a. and b. It is best to teach a and b together so that they are not considered isolated skills. These standards provide the foundation for all future work with linear equations. Students solve equations that have one, zero, or infinitely many solutions and relate those solutions to the context. If the solution is in the form x = a, there is only one solution. If a = a there are infinitely many solutions. If a = b results (where a and b are different numbers), there are no solutions. Linear equations can have fractions and decimals as coefficients and can be solved by expanding expressions with the distributive property and /or collecting like terms. What the **teacher** does: • Provide pairs of students with three one-variable linear equations: One equation has no solution, one has no solutions, and one has an infinite number of solutions. After students have a chance to solve, facilitate a discussion on the results found. Demonstrate how when the result is x = a, there is only one solution that will make the equation true. Use substitution when student results are a = b (a); ask the students to make sense of the results. Ask questions such as, can a = b? Analyze the equation to see why there are no possible solutions. For results a = a, use	8.EE.7**.** As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions. When the equation has one solution, the variable has one value that makes the equation true as in 12 - 4*y* =16. The only value for y that makes this equation true is -1. When the equation has infinitely many solutions, the equation is true for all real numbers as in 7*x* + 14 = 7(*x*+2). As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution. When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in 5*x* - 2 = 5(*x*+1). When simplifying this equation, students will find that the solution appears to be two

substitution to demonstrate how there are infinite number of solutions.

- Present students many opportunities to solve linear equations, including those with fraction, decimal, and positive/negative coefficients. Some equations should provide the opportunities to use the distributive property to expand terms and to combine like terms. Use algebra blocks or tiles to demonstrate combing like terms and using distributive property.

numbers that are not equal or -2 = 1. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution.

Examples:

- Solve for x:
  - $-3(x+7)=4$
  - $3x-8=4x-8$
  - $3(x+1)-5=3x-2$
- Solve:
  - $7(m-3)=7$
  - $\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$

<u>What the **students** do:</u>

- Solve and analyze one variable linear equations and explain whether the solution has one, zero, or infinitely many solutions.
- Solve linear equations with rational coefficients. Use the distributive property when appropriate and combine like terms when the equation calls for it.

<u>Misconceptions and Common Errors:</u>

A common error students make involves applying the distributive property when negative integers are involved, such as -2(x-4). The error occurs when they try to multiply the -2 and the -4. Students need repeated exposure to equations of this type. Prompting students to consider minus 4 or plus negative 4 helps correct the misconception. Providing and discussing tasks that involve students analyzing errors helps students self-correct many misconceptions.

8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

For this standard students will compare the properties of functions. One property of functions is slope. When students are given two different functions, each represented in a different form (algebraically, graphically, in a table, or by a verbal description), students should be able to determine which function has the greater slope. An example follows:

Ruth starts with a $40 gift card for Wal-Mart. She spends $5.50 per week to buy cat food. Let y be the amount left on the card and x represents the number of weeks.

x	y
0	50
1	44.50

2	39.00
3	33.50
4	28.00

Boyce rents bikes for $5 an hour. He also collects non-refundable fee of $10.00 for a rental to cover wear and tear. Write the rule for the total cost (c) of renting a bike as function of the number of hours (h) rented.

Solution: Ruth's story is an example of a function with negative slope. The amount if money left on the card decreases each week. The graph has a negative slope of -5.5, which is the amount the card balance decreases every time Ruth buys cat food.

Boyce's bike rental is an example of a function with a positive slope. This function has a positive slope of 5, which is the amount to rent a bike for an hour. An equation for Boyce's bike could be
$c = 5h + 10$.

<u>What the **teacher** does:</u>

- Present two different linear functions using the same representation (algebraically, graphically, in a table, or by verbal description). Ask the students if they can explain which has the greater slope (rate of change).
- Present two functions each represented in a different form and ask the students to work in groups to determine which has the greater slope. They may need some time to work in groups to change the representation of the functions. Have groups present their answers to the class along with their reasoning. Facilitate the discussion with questions such as, How did you determine which slope is greater? Why did you select to represent the functions in a different form?
- Present two different functions in similar context so that the question about comparing the slopes has meaning.

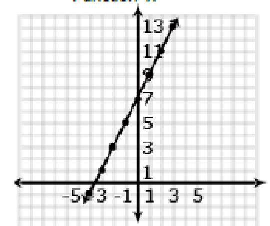
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Note:

8.F.2. Examples:

- Compare the two linear functions listed below and determine which equation represents a greater rate of change.

Function 1:



Function 2:

The function whose input x and output y are related by

$y = 3x + 7$

- Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card

Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks.

x	*y*
0	20
1	16.50
2	13.00
3	9.50
4	6.00

Function 2:

The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).

Solution:

Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the

Function notation is not required for Grade 8.

This standard is the students' introduction to functions and involves the definition of function as a rule that assigns to each input exactly one output. Students are not required to use or recognize function notation at this grade but will be able to identify functions using tables, graphs, and equations.

A relationship is not a function when there is more than one y value associated with any x value. Using the definition, an example of a table that does not represent a function is as follows:

x	y
2	3
1	4
-1	3
2	5

Not a Function

X	y
0	1
1	3
-1	-1
0	1

Function

What the Teacher does:

- Provide graphs of relationships, some of which are functions and some not. Each graph should have a context so that students can reason whether or not the graph makes sense. For example, it does not make sense that a plane can be at different heights at the same point in time. Display this graphically and in a table to see that it is not a function.
  Allow students to make sense of other graphs where a rule assigns to each input exactly one out put.

- Present students with tables of relationships, some of which are functions and some are not. Encourage students to reason whether the example is a function or not and justify their conclusion. Do not limit examples to linear relationships.

- Compare graphs of functions and non-functional relationships with their graphs. Discuss what students notice.

amount the gift card balance decreases with Samantha's weekly magazine purchase. Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$.

What the **students** do:

- Compare properties of functions presented in the same and different forms.
- Communicate the reasoning involved in comparing two functions using precise mathmatical language.

Misconceptions and Common Errors:

A common error students make when working with slopes in context understands what the slope represents. If students are having this problem, work with a single function in a context and then, after identifying the slope and its meaning, add a second function in the same context so that students can work a with the second slope separately before comparing the first slope.

8.F.1. For example, the rule that takes *x* as input and gives x^2+5x+4 as output is a function. Using y to stand for the output we can represent this function with the equation $y = x^2+5x+4$, and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as $f(x) = x^2+5x+4$.

8.F.3. Interpret the equation *y = mx + b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

In this standard students become familiar with the equation y = mx + b as defining a linear function that will graph as a straight line. Students distinguish between linear (functions that graph into a straight line) and nonlinear functions (functions that do not graph into a straight line such a curve). Note that standard form and point-slope form are not studied in this grade.

<u>What the Teacher does:</u>

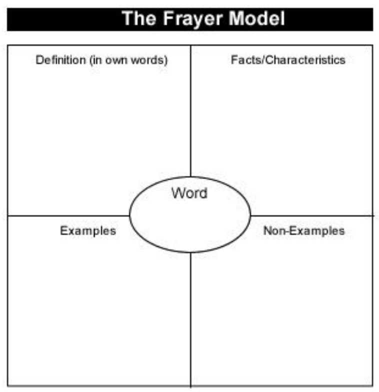
- Present students with examples of functions that are linear and nonlinear for them to graph. Facilitate a class discussion about the similarities and differences in the graphs. The graphs that are not linear are those with points not on a straight line. The area of a square as function of its side length, $A = s^2$, is an example of a nonlinear function because points (1,1), (2,4) and (3,9) are not on a straight line.
- Present a series of linear equations such as the following:

$$y = \frac{1}{2}x + 7$$
$$y = -4x + 8$$
$$y = 6x - 2$$
$$y = 0.5 + 5$$

  Ask students to find the similarities and differences among the equations and their graphs. Facilitate a discussion that results in students recognizing the structure and naming y=mx + b as the general equation for a linear function. Point out that when using a graphing calculator, the general equation for a line is usually expressed as y = ax + b.
- Present some linear equations in the form y = b + mx as many contextual problems will present information in this order. Ask students to write examples of linear functions. This may be a group challenge allowing groups to present their work to the class using correct terminology.

<u>What the Students do:</u>

- Reason whether a table or graph models a function or not and defend their reasoning.
- Use an advance organizer such as the Frayer model to clarify the definition of the function.



<u>Misconceptions and Common Errors:</u>

Students sometimes confuse the terms input and output, knowing that each input can have only one output. Function machines may help these students see that if you put in (input) a number in the machine, the rule only allows one number

8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Students identify that rate of change (slope) and y-intercept (initial value) from tables, graphs, equations, and verbal descriptions of linear relationships. The y-intercept is the y value when the x value is 0. Interpretation of slope and the initial value of the functions is accomplished using real-world situations.

What a Teacher does:

- Present students with graphs of linear functions and focus a discussion on the y-intercept. From examples, lead students to discover that the y-intercept is the y value when the x value is 0. Provide students with opportunities to identify the y-intercept on several graphs.
- Pose the following challenge: Show a table for each of the graphs recently presented and identify the y-intercept in the table. For the table below, the y-intercept is (0,4).

X	Y
-2	-2
0	4
1	7

- Ask students to find the equations for the linear functions previously used and see if they can figure out how to find the y-intercept when the function is in equation form. (It is

to be put out (output). Students can make or draw their own function machines.

8.F.3. Example:

• Determine which of the functions listed below are linear and which are not linear and explain your reasoning.

- $y = -2x^2 + 3$ non linear
- $y = 2x$ linear
- $A = \pi r^2$ non linear
- $y = 0.25 + 0.5(x - 2)$ linear

What the Students do:

- Discern the similarities and differences between linear and nonlinear graphs.
- Look for and make use of structure in identifying y = mx + b as the general form of an equation for a straight line.
- Model functions that are nonlinear and explain, using precise mathematical language, how to tell the difference.

Misconceptions and Common Errors:
Some students have difficulty with the general equation y = mx + b for equations presented as subtraction such as y = 5x – 4. Students can be asked to graph a series of such equations to convince themselves that they are linear. In addition, point out that minus 4 is the same as adding -4.

the constant in the equation y = mx + b) Present some equations where the format is
y = b + mx. Present some equations where the y-intercept is negative.

- Provide context as much as possible so that students learn to interpret the meaning of the initial value in a function.
- Explain slope of a line by presenting a graph of a linear equation and introducing the slope as the ration of the change in they y values of two points to the change in the x value of the same two points. Relate this back to unit rate fro Grade 6.
- Display tables for students to use to determine rate of change using the rise to run ration, such as the following:

X	Y
-2	-2
0	4
1	7

Select any points for example (-2,-2) and (1,7). The difference between -2 and 7 is -9.

- Provide students with the opportunity to discover that coefficient of x in the equation y = mx + b is the slop by allowing them to look at the tables, calculate slope, and compare to the equations of the lines.
- Provide context as often as possible so that students can interpret the meaning of the slope in a given situation.
- Provide students verbal descriptions of situations where they can create the equation of the function. Identify the slope and initial value and relate them to the y= mx + b general equation.

8.F.5.Describe qualitatively the functional relationship between two quantities by analyzing a graph, (e.g. where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described

8.F.4. Examples:

- The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, *c,* as a function of the number of days, *d*.

Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

Days (*d*)	Cost (*c*) in dollars
1	70
2	115
3	160
4	205

Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.

- When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more verbally.

Given a graph, students will provide a verbal description of the function, including whether the graph is linear or nonlinear or where the function is increasing or decreasing. Given a function's verbal description, students will be able to sketch the graph displaying qualitative properties of that function.

What a Teacher does:

- Model the use of mathematical vocabulary to describe the parts of the graph that are linear, increasing, decreasing, and so on.
- Present students with a graph and ask them to tell/write the story and label the axes. A classic example is to write a story about the height of the water in a bathtub over time to match a graph.
- Provide students with opportunities to sketch graphs given the stories.
- Allow students to create their own graphs and stories to share with the class.
- Select stories where the graph may appear counterintuitive such as the graph of plane's distance from its destination city to its time in the air. This graph has a negative slope since as the time increases, the distance to the destination decreases.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the

quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t – 100$ shows the relationship between the time of the ascent in seconds (*t*) and the distance from the surface in feet (*d*).
  - o Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?
- Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

What the Students do:

- Discover the y-intercept (initial value of a function) from a function represented in table, graph, algebraic form and by verbal descriptions.
- Calculate slope of a line using the rise to run ratio.
- Discover slope of a line when the function is presented in a table, graph, algebraic (equation) form, or by verbal descriptions.
- Communicate the meaning of the slope and y-intercept in a given situation using precise mathematical vocabulary.

Student Misconceptions and Common Errors:

The most common error students make is confusing the rise and run in the ration for slope. This mistake is easily observed as students calculate slope. Vocabulary foldables using the terms rise and run may help students remember the differences.

equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

This standard has students solving simultaneous linear equations. It is explained by 8.EE.8a-c. It is best to consider a, b, and c together as they are not isolated skills.

Students will understand the points of intersection are the solutions to pairs of simultaneous linear equations (also know as systems of linear equations). Students will solve systems graphically, algebraically, and by inspection. Examples in this standard are in real-world contexts and mathematical problems.

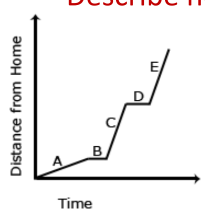
What a Teacher does:

- Have students graph two linear equations that share a solution on the same coordinate plane. The lines should intersect. Facilitate a class discussion about what the point of intersection means. Ask students, what do you notice about the lines? What are the coordinates of the intersection? Which equation does the point of intersection satisfy? When students realize that the point of intersection satisfies both equations, introduce the terms system of equations/simultaneous equations and graphing as a method to estimate solutions to systems of equations.
- Facilitate a discussion about what the graph would look like if there were not solutions to the system of equations and if there are an infinite number of solutions.

8.F.5. Example:

- The graph below shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school.

Describe how each part A-E of the graph relates to the story.



What the Students do:

- Model stories with graphs and vice versa.
- Interpret paths of a story to coincide with parts of the function displayed on a graph.
- Sketch a graph that shows the qualitative features of a function described verbally
- Create a story that matches the qualitative features of a given graph.

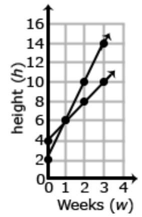
• Provide students with simple cases of simultaneous equations that have no solution and ask them to analyze the equations for a solution. A simple example is, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. • Introduce solving a system of linear equations algebraically and checking the results. • Present opportunities to solve real-world and mathematical problems that are solved by systems of equations. Encourage students to use the most efficient method (graphing, inspection, or algebraic manipulation) to find the solution.	<u>Student Misconceptions and Common Errors:</u> A common error students make is that they do not read the labels on the axes carefully. Eight graders who sketch graphs that appear counterintuitive from the story are making assumptions about the axes without analyzing them. These students should be asked to describe what the axes mean on a graph before they begin to analyze or write a story. 8.EE.8 Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. A system of linear equations whose graphs meet at one point (intersecting lines) has only one solution, the ordered pair representing the point of intersection. A system of linear equations whose graphs do not meet (parallel lines) has no solutions and the slopes of these lines are the same. A system of linear equations whose graphs are coincident (the same line) has infinitely many solutions, the set of ordered pairs representing all the points on the line. By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions. Examples: • Find x and y using elimination and then using substitution. $3x + 4y = 7$ $-2x + 8y = 10$ • Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same. Let W = number of weeks Let H = height of the plant after W weeks

Plant A		
W	H	
0	4	(0,4)
1	6	(1,6)
2	8	(2,8)
3	10	(3,10)

Plant B		
W	H	
0	2	(0,2)
1	6	(1,6)
2	10	(2,10)
3	14	(3,14)

Given each set of coordinates, graph their corresponding lines.

Solution:



Write an equation that represent the growth rate of Plant A and Plant B.

Solution:

Plant A $H = 2W + 4$ Plant B $H = 4W + 2$

• At which week will the plants have the same height?
Solution:
The plants have the same height after one week.
Plant A: $H = 2W + 4$ Plant B: $H = 4W + 2$
Plant A: $H = 2(1) + 4$ Plant B: $H = 4(1) + 2$
Plant A: $H = 6$ Plant B: $H = 6$
After one week, the height of Plant A & B are both 6 inches.

<u>What the Students do:</u>

- Reason that the intersection of two lines on a graph represents the solution to they system of linear equations. Explain, using clear and precise mathematical language, why this is true. Explain how to recognize if the solution set has one, zero, or infinite number of points.
- Solve systems of equations graphically, algebraically, and by inspection depending on the problem presented.
- Solve real-world and mathematical problems that lead to pairs of simultaneous linear equations.

	Student Misconceptions and Common Errors: Common errors for systems of equations include students who have trouble accurately graphing and, therefore, cannot correctly estimate the solution. Technology can be helpful as can graph paper with larger than 1-cm squares.
Standards for Mathematical Practice	
Understand the connections between proportional relationships, lines, and linear equations. **8.EE.5, 8.EE.6** In this cluster students connect proportional relationships, lines, and linear equations. First, students compare proportional relationships represented in different ways such as graphs, tables, and linear equations. Unit rate is interpreted as the slope of a line, and students learn that the slope is the same between any two points on a line by using similar triangles. Then the general equations for a line (y + mx + b and y = mx) are derived. **MP2. Reason abstractly and quantitatively.** **MP6. Attend to precision.** **MP7. Look for and make use of structure.** **Define, evaluate and compare functions.** **8.F.1, 8.F.2, 8.F.3** Students are introduced to functions as rules that assign exactly one output to each input. Functions are represented graphically algebraically, numerically in tables, and by verbal descriptions. Function notation (f(x)) is not required in Grade 8. Students compare properties of two functions represented in different forms such as determining which function has a greater rate of change from two functions, one represented graphically and one numerically n tables. Students recognize equations in the form of y = mx + b as defining linear functions as opposed to those that are none linear (quadratic, exponential). **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP5. Use appropriate tools strategically.**	Students compare two proportional relationships represented in different forms. Students give explanations that are precise and use appropriate vocabulary. Students see a pattern that results in the general form of a linear equation.

MP7. Look for and make use of structure. **Use functions to model relationships between quantities.** **8.F.4, 8.F.5** In this standard students use what they have learned previously and apply it in context to model functional relationships. Students construct functions and determine the slope and y-intercept (initial value) of a function from a verbal description of a relationship or from two (x, y) values, including finding those values in a graph or a table. Students give contextual meaning to the rate of change and y intercept and interpret rate of change and y intercept in terms of the graph or table of the function. Give a graph, students analyze the functional relationship (does the function increase or decrease? Is it linear?) Given a verbal description of a function, students sketch the function showing the qualitative features. **MP4. Model with mathematics.** **MP7. Look for and make use of structure.**	Students determine if a relationship is a function. Students represent linear function in algebraic, graphical, numerical, and verbal forms. Students use technological tools to explore and deepen their understanding of functions. Students apply general mathematical rules such as y = mx + b as the equation for a linear function.
Analyze and solve linear equations and pairs of simultaneous linear equations. **8.EE.7, 8.EE.8** Students analyze and solve one variable linear equations for one, zero or infinitely many solutions, simplifying the equations until they reach x = a, a = a, or a = b) where a and b are different numbers). Students then apply that knowledge to analyzing and solving pairs of simultaneous linear equations also known as systems of linear equations in two variables. **MP1. Make sense of problems and persevere in solving them.** **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP6. Attend to precision.** **MP7. Look for and make use of structure.**	Students construct a function to model a linear relationship between two quantities. Students make use of the qualitative features (structure) found in a verbal description of a function and sketch that function.

	Students solve problems with systems of linear equations. Students analyze linear equations and systems of linear equations. Students model real-world problems with systems of equations. Students give explanations that are precise and use appropriate vocabulary. Students use structure of an equation to know which steps to perform to solve it.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*

• Input/output tables can be used as a tool to generate a function rule.
• A graph of a function is the set of ordered pairs consisting of an input and its corresponding output
• Functions can be represented algebraically, graphically, numerically in tables (ordered pairs), or by verbal descriptions.
 	
• Changing the way a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations may be more useful than others and may highlight different characteristics.
• Some representations of functions may show only part of the function.
	• GRAPH (proportional relationships)
	• INTERPRET (unit rate as slope)
	• COMPARE (different representations of proportional relationships)
	• EXPLAIN (why slope is the same between any two points on a non-vertical line)
	• DERIVE (linear equations ($y = mx$ and $y = mx + b$)
	
	• SOLVE (linear equations including equations with fractional number coefficients)
	• GIVE (example of linear and nonlinear equations)
	• EXPAND (expressions and equations by)
	  ○ Use (distributive property)
	  ○ Collect (like terms)

• Functions are used to model real-world phenomena.
• Proportional relationships can be represented symbolically (equation), graphically (coordinate plane), in a table, in diagrams, and verbal descriptions.
• In a proportional linear relationship, the point (0, 0) is the y-intercept and (1, *r*) is the slope, where *r* is the unit rate.
• Slope of a line is a constant rate of change.
• The y-intercept is the point at which a line intersects the vertical axis (y-axis).
• One form of an equation for a line is $y=mx + b$, where *m* is the slope and b is the y-intercept. A special case of linear equations (proportional relationships) are in the form of $y/x = m$ and $y = mx$.
• Inverse operations are used to solve equations.
• Linear equations in one variable can have one solution, infinitely many solutions, or no solution.
• A graph of a linear function is a straight line.
• A function is a rule that assigns to each input exactly one output.
• Functions describe situations where one quantity determines another.
• Equations in one variable can have one solution, no solutions or infinite solutions.
• The formula Distance = Rate × Time
• Linear Equations (Simultaneous/system of)
  ○ Rational Number Coefficients
  ○ One variable
    ▪ One solution
    ▪ Many solutions
    ▪ No solutions
	• UNDERSTAND (function is a rule)
	  ○ GRAPH (sets of ordered pairs)
	• COMPARE (functions)
	  ○ Algebraically
	  ○ Graphically
	  ○ Numerically in tables
	  ○ Verbal descriptions
	• CONSTRUCT (function)
	  ○ Model (linear relationship using table of values or graph)
	• DETERMINE (rate of change and initial value of function)
	• INTERPRET (table or graph)
	• DESCRIBE (functional relationship between two quantities)
	• SOLVE (multi-step equations including distributive property, combining or collecting like terms and with variables on both sides of the equations)
	• CREATE (graph from a verbal description)
	• SOLVE (linear equations)
	• SHOW (simpler forms)
	• ANALYZE (linear equations)
	• UNDERSTAND (solutions)
	• SOLVE (systems of equations)
	• ESTIMATE (solutions)
	• GRAPH (equations)
	• SOLVE (simple cases by inspection)
	• CONSTRUCT (function)
	• READ (table and graph)
	• INTERPRET (rate of change and initial value of a linear

• Equations into simpler forms
  ○ Expanding Expressions
  ○ Distributive Property
  ○ Combining Like Terms
 	
• Function
  ○ Rate of change
  ○ Initial value (of a linear function)
  ○ Representation
    ▪ Algebraically
    ▪ Graphically
    ▪ Numerically in tables
    ▪ Verbal description
	function

UNDERSTAND

Big ideas, generalizations, principles, concepts, ideas that transfer across situations

There are different families of functions (linear and nonlinear) that can be used to model different real-world phenomena.

The connection between linearity and proportionality (as a special case of linearity) is based on an understanding of slope as the constant rate of change and the y-intercept.

The meaning of the solution to a system of equations.

Common Student Misconceptions for this Unit

• Students may not be aware that they can use the inverse of the fraction to simplify the equation. • Students may not recognize when the coefficient is a -1 • Students may multiply/divide first instead of adding or subtracting first. • Students may not recognize that the subtraction sign in front of a coefficient will make that coefficient negative. • Students may struggle if they do not have a deep understanding of integers • Students may not distribute to every term in the parentheses. • Students may struggle with the organization and precision of their work. • Students may combine unlike terms. • Students may combine terms across the equal sign. • Students may confuse the x and y axis. • Students may switch the rise and the run. • A common problem when students learn about the slope-intercept equation $y = mx + b$ is that they mechanically substitute for m and b without understanding their meaning. • Students who are still concrete thinkers will struggle with making the connection of real life information and graphing. • Students may not recognize the difference between positive and negative slopes. • Students may not recognize the placement of the negative sign in a fraction may be placed in front, with the numerator or with the denominator. • Students will interchange positive and negative intercepts. • Students may not check their work to validate the solution to the linear equation. • Students may not solve for the second variable when solving a system of equations. • Students may not substitute the whole expression when solving by substitution.

Unit Assessment/Performance Task	DOK
• Unit 3A Test Linear Relationships • Unit 3A Test Solving Linear Equations • Unit 3B Test Solving Systems of Linear Equations • Meals Out	

• Picking Apples • Baseball Jerseys • Linear Graphs • Bike Ride • The Journey	

Vocabulary
• Combine (Collecting) Like Terms • Common term • Consistent (one solution) • Dependent System of Equations (infinite solutions, same slope, and y-intercept) • Distributive Property • Elimination Method (also known as Linear Combination) • Equation • Equivalent • Evaluate • Exponents • Expression • Factor • Function • Graphical Method • Identity • Inconsistent (no solution), Inconsistent System (no solution) • Initial Value of a Function • Input/Output • Irrational Number • Inverse Operations

- Linear relationship
- Non-Linear
- Numerical Expressions
- Ordered Pairs
- Point of intersection
- Proportional Relationships
- Rate of Change
- Rational Numbers
- Rise
- Run
- Similar Triangles
- Slope
- Slope-Intercept Form
- Standard form
- Substitution Method
- System of Equations
- Unique solution
- Unit Rate
- Variable
- Y-intercept

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons. **Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)**

For lessons 1-5, Algebra tiles can be used to represent the equations and aid students to solve equations.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=216 *http://www.jamesrahn.com/homepages/algebra_tiles.htm*

Lesson 1: Solving Multi-Step Linear Equations Using Distributive Property

Students will be given teacher created sets completed problems. Some of the problems are accurately solved and the rest have a variety of flaws. They will decide which are correct and which are incorrect and then they will have to explain why the provided work is correct or flawed. Justify their answers.

- Solve multi-step linear equations with rational number coefficients

Lesson 2: Solving Multi-Step Linear Equations, Expanding Expressions by Using Distributive Property and Combining or Collecting Like Terms on one side of equation

Students will receive a blank answer sheet. 20 questions will be posted around the room for each student to complete. Students will have (depending on the level) a certain amount of time to finish each problem. After the time is up, students need to move on to the next question.

- Solve multi-step linear equations, collecting like terms on one side of equation, with rational number coefficients
- Solve multi-step linear equations using distributive property and collecting like terms on one side of equation, with rational number coefficients
- Examples can be found at: (cut and paste website)

http://www.mhhe.com/math/precalc/barnettpc5/graphics/barnett05pcfg/ch01/others/bpc5_ch01-01.pdf

Lesson 3: Solving Multi-Step Linear Equations Using Distributive Property and Combining Like Terms on both sides of equation (2 days)

Each student will receive an index card with an equation on it. They will then complete one step and pass the card to the next student. Once the student receives the card, they will have to determine if *the previous step is correct or not and then complete the next step.*

- Solve multi-step linear equations using like terms on both sides of equation, with rational number coefficients
- Solve multi-step linear equations using distributive property and like terms on both sides of equation, with rational number coefficients
- Solve Linear Equations with One Solution, Infinitely Many Solutions, and No Solutions (Inconsistent Equation)
    - Solve linear equations with one solution so that x=a where a is a real number (Ex: 12 – 4x = 16. The only value for x that makes this equation true is -1)
    - Solve linear equations with infinitely many solutions so that a=a where a is a real number (Ex: 7x + 14 = 7(x + 2). As the equation is simplified and solved, variable terms are eliminated, leaving 14=14 or 0=0)
    - Solve linear equations with no solutions so that a=b where a and b are different real number (Ex: 5x – 2 = 5(x -+ 1). As the equation is simplified and solved, the solution is two numbers that are not equal, -2=1)
    - Once all of the equations have been solved, students can work in groups to write word problems that would use the equations to be answered

Lesson 4: <u>Solving Equations with no solution or infinite solutions</u>

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

Lesson 5: <u>Proportional Relationships-Unit Rate</u>

Students will use the Navigating Through Algebra Series "Pledge Plans" to analyze and graph *different pledge plans. See Math08_unit04_LA06*

- Model a linear equation with a graph (graph proportional relationships using table of values)
- Interpret unit rate(the proportional relationship) as the slope of the graph

Lesson 6: Comparing Proportional Relationships

Students will use the Navigating Through Algebra Series "Walking Strides" experiment with different walking rates. See math08_unit04_LA07

Using the following web link students learn to read distance time graphs and to derive basic information about rate from the graphs. All graphs involve constant speed. (cut & paste website) *http://www.teachersdomain.org/resource/vtl07.math.data.rep.lpgraphdis/*

- Compare different distance-time graphs to determine which represents a greater speed (including a description of the unit rate in the explanation)
- Compare two different distance time relationships represented in different ways ((including a description of the unit rate in the explanation)

Lesson 7: Explaining the Slope of a Line

- Identify similarity of two right triangles graphed on a coordinate plane (passing through the origin and explain same slope between any two points on a non-vertical line (the hypotenuse of the right triangles) (8.EE.6 Example)
- Express each line as an equation

Lesson 8: Representing Linear Patterns

- Represent linear data patterns in familiar real world contexts as an equation (with different positive and negative slopes and different y-intercepts, including the origin).
- Graph the equations

- Derive the linear equations y=mx and y=mx + b (for a line intercepting the y-axis at b)
- Give examples of linear equations.

Lesson 9: Represent Functions with Positive and Negative Slopes Algebraically, Graphically, Numerically, and by Verbal Description

Students will perform a matching game.

- Represent linear functions with positive and negative slopes and different y-intercepts algebraically, graphically, numerically, and/or by verbal description
- Compare properties of two different functions, each represented in a different way, including which has the greater rate of change, which has a negative slope, etc.

Lesson 10: Understanding and Interpretation of Linear Patterns as Functions

Students will be provided with graphing calculators. They will then be provided with equations that they need to graph and determine if they are linear or non-linear equations. This will lead to students discovering the characteristics of non-linear equations.

- Understand and represent a linear equation and graph as a function, a rule with x as the input and y as the output
- Interpret y=mx + b as a linear function whose graph is a straight line
- Compare and determine which functions are linear or non-linear

Lesson 11: Constructing and Modeling Functions

An available resource for various graphing and modeling activities can be found at:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/exploring-linear-relationships

- Construct a function, modeling a linear relationship between two quantities
- Determine the rate of change and initial value of the function from an equation, a table, a graph, and a verbal description
- Interpret the rate of change and initial value of the function from an equation, a table, a graph, and a verbal description

Supplemental Materials and Resources

8.EE

- NYC Unit on EE- Extends the ideas more formally into the realm of algebra: http://schools.nyc.gov/NR/rdonlyres/835F949D-A3D9-419E-A54C-A004209AAC80/0/NYCDOEG8MathExpressionsandEquations_Final.pdf

Understand the connections between proportional relationships, lines, and linear equations.

- Students will interpret a distance/time graph describing a bike ride: http://map.mathshell.org.uk/materials/tasks.php?taskid=363#task363
- Students will read a description of a car journey and draw a distance-time graph to represent it: http://map.mathshell.org.uk/materials/tasks.php?taskid=373#task373
- Students must figure out how many planks and bricks are needed to build a bookcase: http://map.mathshell.org.uk/materials/tasks.php?taskid=382#task382

Analyze and solve linear equations and pairs of simultaneous linear equations.

- Students will look at a number of equations and inequalities that have more than one solution: http://map.mathshell.org.uk/materials/tasks.php?taskid=263#task263
- Students will write equations to solve problems about buying bags of chips and candy bars: http://map.mathshell.org.uk/materials/tasks.php?taskid=366#task366
- Students will compare two methods of converting temperature measurements from Celsius to Fahrenheit: http://map.mathshell.org.uk/materials/tasks.php?taskid=388#task388

8.F

- A set of short tasks: http://map.mathshell.org.uk/materials/tasks.php?taskid=400#task400

Define, evaluate, and compare functions.

- Students are asked to match equations with linear graphs: http://map.mathshell.org.uk/materials/tasks.php?taskid=374#task374
- Students will be able to verbalize the meaning of the equation to reinforce understanding and discover that slope (or rate of movement) is the same for all sets of points given a set of data with a linear relationship.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L769

Use functions to model relationships between quantities.

- Students will help Bill to find the best price for buying printed jerseys for the baseball team: http://map.mathshell.org.uk/materials/tasks.php?taskid=362#task362
- Students use equations to solve a problem with a restaurant check: http://map.mathshell.org.uk/materials/tasks.php?taskid=376#task376
- Students are asked to match equations with linear graphs: http://map.mathshell.org.uk/materials/tasks.php?taskid=374#task374

The following link demonstrates representing patterns in multiple ways.

http://www.edu.gov.on.ca/eng/studentsuccess/lms/files/tips4rm/gr8Unit2.pdf

Interdisciplinary Connections:

- Students can find real life graphs from companies and then compare them.
- Science
    - Recording and graphing examining data and then analyzing the graph for information
    - Calculating and graphing speed, distance, time relationship
    - Use of formulas
- Social Studies

- Interpreting real world graphs (growth of population, birth rate)

Literature connection:

- **Anno's Magic Seeds Lesson Plan –** set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graphs http://www.scholastic.com/teachers/lesson-plan/annos-magic-seeds-lesson-plan
- "The Adventures of Penrose the Mathematical Cat" By: Theoni Pappas
- "Mathographics" By: Robert Dixon

Tools/Manipulatives
Chart paper Cubes Graph paper Graphing calculators Number Line Square tiles

Suggested Formative Assessment Practices/Processes
• Problem of the Day • Lesson Quizzes • Entrance and Exit Slips • Anecdotal Records (Topic Observation Checklist)

Differentiation and Accommodations
• Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments